Controller Design Based on Fractional Filter with IMC-PID: Application to Servo System and Single Area Power System

Vivek Kumar
Department of Electrical Engineering
Indian Institute of Technology, Roorkee
Uttarakhand, India
vivek_k1@ee.iitr.ac.in

Yogesh V. Hote
Department of Electrical Engineering
Indian Institute of Technology, Roorkee
Uttarakhand, India
yogesh.hote@ee.iitr.ac.in

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***Abstract*—The Proportional-Integral-Derivative (PID) controller tuned by Internal-Model-Control (IMC) is extensively used in industrial control applications. This methodology offers an excellent trade-off between the setpoint tracking and disturbance rejections, and also provides better robustness. In this paper, we suggest a simple and straightforward approach for designing IMC based PID controller with a fractional filter for electrical engineering applications. Due to the use of a fractional order filter, the flexibility of tuning the parameters is increased. To verify the suggested method's utility, simulation analysis has been done for the mathematical model of a rotational DC servo system (QUBETM-Servo 2) and single area power system (SAPS). The approach for controller design depicts proper set-point tracking and better disturbance rejections. The performance analysis of the controller which has been designed for the applications has been done based on the integral of error (IE_E), integral square error (ISE_E), integral absolute error (IAE_E), and control efforts (CE). Finally, the robustness analysis has been done for a +50% change in the gain of the process.**

***Index Terms*—Internal-Model-Control, PID, Fractional Filter, Robustness, Performance**

I. INTRODUCTION

Proportional-Integral-Derivative (PID) controllers have been extensively used in the last few decades in the area of control and process control for wide-ranging applications (including the industrial equipment, food and chemical sector, automobile engineering, mining, electric power sector, and aerospace factories). This field of study is still very active today [1]. PID controller design is important because of the its simplicity, and easily implementable structure, which typically results in various tuning methods and implementation options. These tuning methods are based on quick set-point tracking, good disturbance impact reduction, avoiding overaggressive control action, preventing the integral windup, robust-ability to process variables, and performance index's minimization [2], [3]. PID tuning by using robust optimization (like H_∞ [4]) and soft computing methods (such as evolutionary algorithms and fuzzy logic principles [5], [6]) have been published in recent years. Internal model control (IMC) is a basic, simple, resilient, and easy-to-implement technique that may be used to tune linear, nonlinear, and delayed processes. The development of a PID controller with an internal model control (IMC) structure has been carried out by various researchers significantly [7], [8], [9], [20]. It is observed that in all these papers, the three parameters of PID are obtained with one parameter (which is IMC filter time constant (λ)), and in the case of fractional order filter, there is one more parameter (fractional order of the IMC filter (σ)). In this method, it is seen that the filter arrangement and it's choice are crucial in defining the parameters of the PID controller. Generally, the IMC filter structure has chosen like as [9]:

$$f(s) = \frac{1}{(\lambda s + 1)^p}; \qquad p \,\epsilon\, I \tag{1}$$

where $\lambda -$ is time constant of the filter and $p -$ is the filter's order (integer in nature).
In this paper, we have chosen fractional IMC filter as [9]:

$$f_f(s) = \frac{1}{(\lambda s^\sigma + 1)^p}; \quad 1 < \sigma < 2; \quad p \,\epsilon\, I \tag{2}$$

where $\sigma -$ is the filter order which is fractional in nature.
To achieve optimum PID tuning, experts have demonstrated various IMC approaches. Some schemes include complicated mathematical calculations, while other techniques need an extra filter term along with a conventional PID controller [10]. For example, the IMC-PID approach put forward by Rivera et al. [7], showed that some process models of the first order and second-order, integrating kind of process, changing $Q(s)$ into $C_f(s)$ produces additional filter-term. In this paper, the final controller structure will include an extra filter term with the PID controller. The IMC-based PID controller does not always provide adequate disturbance attenuation (slow response). Hence, tuning rules are constantly being developed to improve disturbance attenuation for FOPDT and SOPTD systems [11]. The inherent disturbance attenuation capability of closed-loop controllers has been studied in the paper [12]. As a result, the current effort is to construct a controller for

FOPDT and SOPTD systems to reject disturbances effectively. With fractional order differentiation and integration methods, the structure allows to use fractional calculus to produce a fractional order controller for efficient performance of closed-loop system [13]. By using a controller with fractional order, the performance can be improved in comparison with an integer order controller. Here, fractional order controllers offer reliable control with extra tuning factors, which increases the tuning complexity slightly. In [14], an ideal IMC filter that minimizes IAE for a particular M_s was found by monitoring the closed loop behavior after developing PID controllers for several process systems. Implementing a controller using a fractional IMC filter for various applications is the primary goal. In paper [15], Authors suggested a fractional IMC-filter of the first order as a PID controller for SOPTD processes based on IMC. The tuning variables in [15], were selected repeatedly to have minimum values for IE, ISE, and IAE. Due to the improvement in the system performance after using the first-order fractional filter, motivated to construct a higher-order fractional filter for more improvement and flexibility. In the present work, a second-order IMC filter with fractional order has been used for single area power system and also DC servo motor for velocity control.

In this paper, the following points have been addressed:

- PID controller design with fractional filter based on IMC structure tuning for the particular value of maximum sensitivity (M_s).
- The trial-error method is used for tuning the filter parameters for a particular value of (M_s).
- Applications of the suggested approach in servo system and single area load frequency control problem.
- The results are analyzed in the presence of load disturbances and parametric uncertainty.
- Evaluation of control scheme in the presence of performance indices such as: IE_E, ISE_E, IAE_E, and CE.

II. INTERNAL MODEL CONTROL (IMC) SCHEME WITH FILTER FRACTIONAL IN NATURE

Fig. 1, depicts a diagram illustration of the IMC framework with a fractional order filter. Whereas reference input, disturbance input, and closed-loop output, are represented by $R_i(s)$, $D_i(s)$, and $Y_o(s)$, respectively. Additionally, a system $G(s)$ is involved in this arrangement, and it is used to construct the controller in combination with the system model $G_m(s)$ [7].

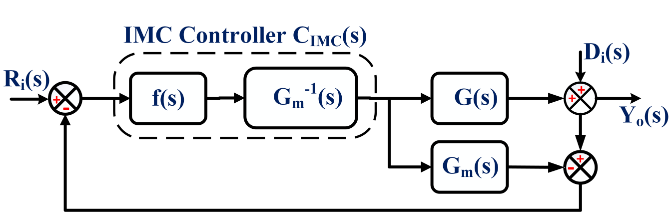

Fig. 1. Fundamental diagram of IMC scheme [7].

To construct the IMC-based PID controller, we need to follow

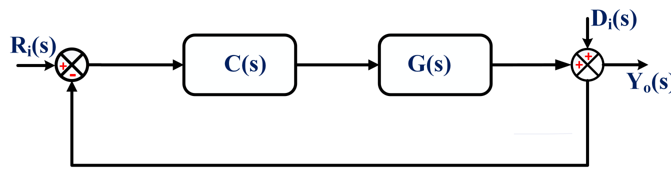

Fig. 2. Fundamental closed loop system structure.

the following steps:

Step 1: The process model is segregated into two parts. (1). Invertible part of the process, (2). Non-Invertible part of the process.

$$G_{pm}(s) = G_{pm}^{+}(s)G_{pm}^{-}(s) \tag{3}$$

where $G_{pm}^{+}(s)$ consists of non-invertible parts of the process like delay time and zeros on the right half plane. $G_{pm}^{-}(s)$ consists invertible part of the process, which has poles and zeros on left half of the plane.

Step 2: To develop the controller design of the IMC, we need to add a low pass filter with invertible part of the process, here the filtering nature is fractional so that the IMC controller is appropriate [7].

$$C_{IMC}(s) = \frac{1}{G_{pm}^{-}(s)} f_f(s) \tag{4}$$

where $f_f(s)$ is a fractional order filter modeled as:

$$f_f(s) = \frac{1}{(\lambda s^{\sigma} + 1)^p} \tag{5}$$

where λ (time constant of the fractional filter) is an adjustable parameter that modifies a closed-loop system's response time and eliminates process/model incompatibility, typically happening at high-frequency regions and contributing to robustness. Furthermore, σ is the fractional order of the filter, i.e., an adjustable variable, which improves the flexibility of the suggested controller, and the range of the σ is between 1 to 2. Here, the value p has been taken to make C_{IMC} strictly-proper/proper for the physical realization of the IMC controller

Step 3: Finally, the closed-loop feedback control formulation is shown in Fig. 2 after simplification, which can be designed as:

$$C(s) = \frac{C_{IMC}(s)}{1 - C_{IMC}(s)G_{pm}(s)} \tag{6}$$

III. MATHEMATICAL FORMULATION OF PID CONTROLLER BY USING FRACTIONAL FILTER

According to Fig. 1, and Fig. 2, the suggested structure for the designed controller is obtained as follows [15]:

$$C(s) = (Fractional\ Filter\ part) \times K_c \left(1 + \frac{1}{\tau_{it} s} + \tau_{dt} s\right) \tag{7}$$

The designed controller configuration consists of the PID controller cascaded with a fractional filter term. Tuning of the fractional filter parameters is based on a heuristic approach

and is enough to optimize the closed-loop response.
Consider the first order transfer function as:

$$G_{pm}(s) = \frac{K_s}{(sT_s+1)} \tag{8}$$

where K_s- is system's gain and T_s- is time constant of system model.
The optimum filter of fractional order has been chosen as given in eq. (5). After using the IMC filter with the invertible part of the process, the IMC control structure is:

$$C_{IMC}(s) = \frac{(sT_s+1)}{K_s(\lambda s^{\sigma}+1)^2} \tag{9}$$

The designed controller based on IMC-PID for closed loop system, as mentioned in eq. (6):

$$C(s) = \frac{\left[\frac{(sT_s+1)}{K_s(\lambda s^{\sigma}+1)^2}\right]}{1-\left[\frac{(sT_s+1)}{K_s(\lambda s^{\sigma}+1)^2}\right]\left[\frac{K_s}{(sT_s+1)}\right]} \tag{10}$$

Finally,

$$C(s) = \left(\frac{1}{\lambda^2 s^{2\sigma}+2\lambda s^{\sigma}}\right)\times\frac{1}{K_s}(1+sT_s) \tag{11}$$

After comparison with ideal form of the PID controller, the parameters values are:
$K_c = \frac{1}{K_s}$; $\tau_{it} = 0$; $\tau_{dt} = T_s$

IV. PERFORMANCE AND ROBUSTNESS ANALYSIS OF THE SYSTEM

To accomplish the desired objectives, we need to analyze the results or responses of the process model for the designed controller methodology. So, the evaluation of the closed-loop performance has been done based on accumulated errors, which are integral of error (IE_E), integral square error (ISE_E), integral absolute error (IAE_E), and control effort (CE) for a fixed value of the maximum sensitivity (M_s) in the response to follow set-point and disturbance attenuation. The performance indices are given below [18], [19]:

$$IE_E = \int_0^{\infty} e_r(t)dt \tag{12}$$

$$ISE_E = \int_0^{\infty} e_r^2(t)dt \tag{13}$$

$$IAE_E = \int_0^{\infty} |e_r(t)|dt \tag{14}$$

$$CE = \sum_{k=0}^{\infty} |u_{k+1}-u_k| \tag{15}$$

$$M_s = \max_{0\le\omega\le\infty}\left|\frac{1}{1+G_{pm}(j\omega)C(j\omega)}\right| \tag{16}$$

The performance of a controller to function effectively in uncertain conditions is the key design objective. In the real universe, every system has some imperfections in its model. So, the performance/efficiency of the designed controller must be verified in the existence of disturbances, noises, and parametric uncertainty. The designed controller should be robust in these conditions. The stability and resilience of a closed-loop control system to process parameter fluctuations are based on the sensitivity function M_s, which is mentioned in (16) and complementary sensitivity functions indicated by M_T mentioned in (18). These requirements are based on the loop function of the Nyquist stability condition. For effective control, it is advised that the maximum sensitivities fall between 1 and 2. The minimum value of maximum sensitivity is defined by the maximum distance between the Nyquist-plot and critical-point $(1+j\omega)$. Therefore, the essential requirement to ensure robust stability can be derived from the small gain theorem [21], [22], which is given below:

$$\epsilon_m(j\omega)M_T(j\omega) < 1, for\ \ all\ \ \omega\in(-\infty,\infty) \tag{17}$$

where $\epsilon_m(j\omega) = \left|\frac{G(j\omega)-G_{pm}(j\omega)}{G_{pm}(j\omega)}\right|$ - is multiplicative bound for plant. To ensure the robustness, there should be an upper bound so that $\epsilon_m(s) < 1$.
And $M_T(j\omega)$ - is complementary sensitivity function as given in eq. (18).

$$M_T(j\omega) = \max_{0\le\omega\le\infty}\left|\frac{G_{pm}(j\omega)C(j\omega)}{1+G_{pm}(j\omega)C(j\omega)}\right| \tag{18}$$

For the robustness condition due to change in the process gain and time delay [16], the suggested design must be verified as per eq. (19), [16].

$$\|M_T(j\omega)\|_{\infty} < \frac{1}{\left|\left(\frac{\delta K_s}{K_s}+1\right)e^{-\delta t_s}-1\right|} \tag{19}$$

V. SIMULATION STUDIES AND DISCUSSION

The analysis of the process models has been carried out in this section of the paper with the help of MATLAB/Simulink (MATLAB R2022a).
Example 1: We applied existing controller technique to address the issue with a DC servo system (QUBETM-Servo 2) made by Canadian company Quanser Inc. The system contains an integrated amplifier with a current sensor module rated at 18 Volt default input voltage, 0.54 Amp current, 3050 RPM normalized speed, 22.0 mN-m normalized torque, and a brushed DC motor with an optical encoder that controls velocity.
The mathematical model of the setup is given below [23]:

$$G_{pm}(s) = \frac{21.721}{(0.147s+1)} \tag{20}$$

After implementing the designed methodology, the tuned parameters of the PID controller are: $K_c = 0.046$, $\tau_{it} = 0$, and $\tau_{dt} = 0.147$. Also, the tuned parameters of the cascaded filter part, as shown in eq. (21) are obtained for the fixed value of maximum sensitivity ($M_s = 1.2$) are: $\lambda = 0.4$sec and $\sigma = 1.0484$, here heuristic approach is used for tuning the parameters for a fixed value of M_s.

$$f_f(s) = \left(\frac{1}{\lambda^2 s^{2\sigma}+2\lambda s^{\sigma}}\right) \tag{21}$$

So, the final control structure is given as:

$$C(s) = \left(\frac{1}{0.16s^{2.0968} + 0.8s^{1.0484}}\right) \times 0.046(1 + 0.147s) \tag{22}$$

To analyze the performance of the designed method, a unit step function is used as the reference input and disturbance input at 15sec. The closed-loop response has been evaluated based on IE_E, ISE_E, IAE_E, and CE, which is recorded in Table. I, for set-point tracking and disturbance rejection. Furthermore, the servo, as well as the regulatory plots, have been observed in Fig. 3, and the plot for observing the effort of the controller has been shown in Fig. 4. From the responses, it can be observed that set-point tracking is fast and smooth with good disturbance rejection. Also, the robust analysis of the process model can be shown in Fig. 5, which proves the robustness of the designed method for $+50\%$ uncertainty in the system's gain. The experimental validation and comparative analysis of this model based on the designed scheme will be carried out as the extension of the work.

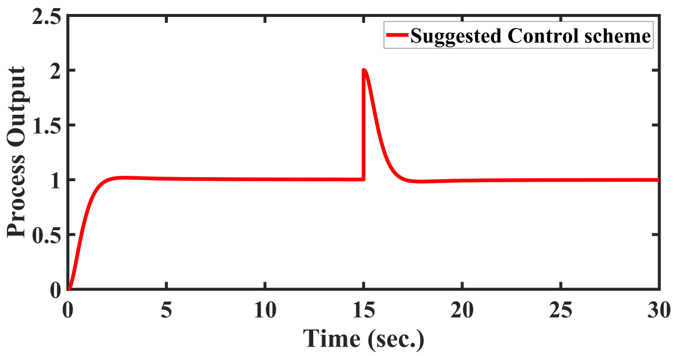

Fig. 3. Process Response of Example 1.

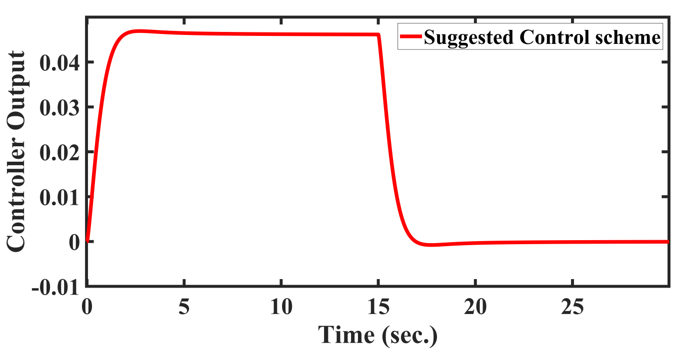

Fig. 4. Control Effort Response of Example 1.

Example 2: The suggested design is also implemented for the load disturbance rejection (load frequency control (LFC) problem) in a single-area power system (SAPS). The SAPS for the LFC model consists of a governor $G_G(s)$, a turbine (non-reheated) $G_T(s)$, load with machine $G_P(s)$, and droop characteristics $1/R$ as shown in Fig. 6. The transfer function

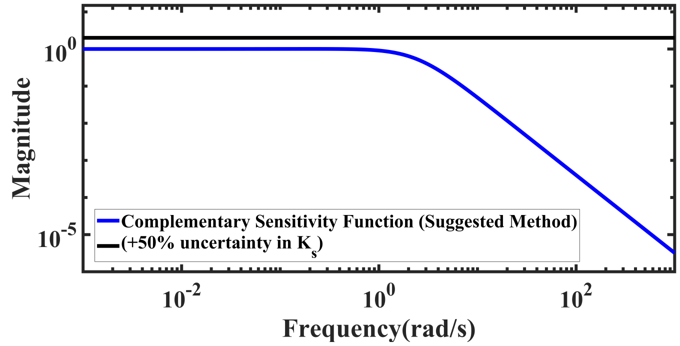

Fig. 5. Robust Response of Example 1.

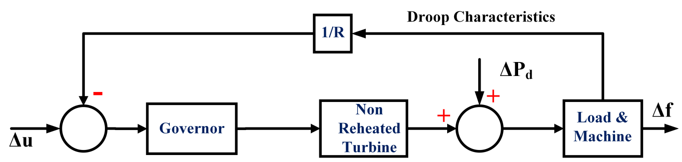

Fig. 6. Single Area Power System Model [24].

model of these systems is [17]:

$$G_G(s) = \frac{1}{sT_g + 1} \tag{23}$$

$$G_T(s) = \frac{1}{sT_t + 1} \tag{24}$$

$$G_P(s) = \frac{K_p}{sT_p + 1} \tag{25}$$

The final mathematical model of the SAPS is given below [17]:

$$G_{pm}(s) = \frac{250}{s^3 + 15.88s^2 + 42.46s + 106.2} \tag{26}$$

After applying the suggested methodology, the tuned values of the PID parameters are $K_c = 0.169$, $\tau_{it} = 0.399$, and $\tau_{dt} = 0.315$, with fractional filter parameters: $\lambda = 0.7$sec and $\sigma = 1.0482$ for a fixed value of $M_s = 1.2$.
So, the final control structure is given below:

$$C(s) = \left(\frac{s}{0.7s^{2.0964} + 1.4s^{1.0482}}\right) \times 0.169(1 + \frac{1}{0.399s} + 0.315s) \tag{27}$$

For evaluating the results of the suggested techniques, a load disturbance of step input with magnitudes -0.02, -0.2, and -0.5 have been used. The results of disturbance rejection have been observed from Fig. 7, which provide a fast and smooth response for step disturbances of different magnitudes. Also, the robust analysis of SAPS has been done for $+50\%$ uncertainty in load gain, which can be seen in Fig. 8, which verifies the suggested technique is robust.

TABLE I
CRITERIA FOR EVALUATING PERFORMANCE

Method	M_s	Set-Point Tracking				Disturbance Rejection			
		IE_E	ISE_E	IAE_E	CE	IE_E	ISE_E	IAE_E	CE
Proposed	1.2	0.6611	0.517	0.8762	0.0477	0.6844	0.5173	0.8528	0.0477

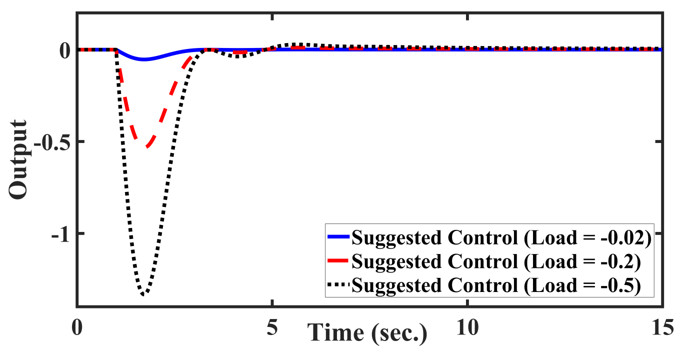

Fig. 7. Load Disturbance Response of Example 2.

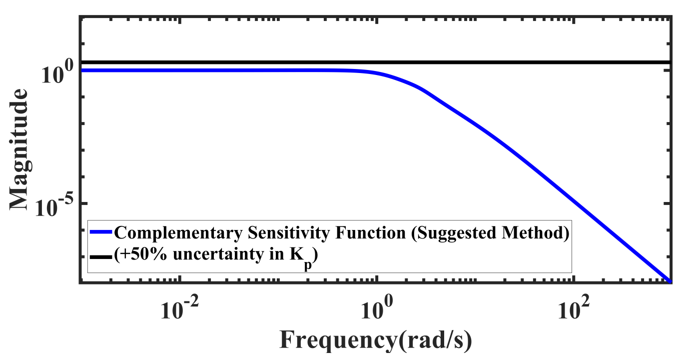

Fig. 8. Robustness Analysis of Example 2.

VI. CONCLUSION

This paper suggests a technique for designing the controller based on IMC structure for PID cascaded with a low pass filter which is fractional in order. The main benefit of the suggested method is that it is simple, straightforward, and easily implementable. The efficacy of the designed controller has been verified by a simulation study of a DC servo system (QUBETM-Servo 2), and the experimental analysis of the same model will be carried out in extended work. The performance evaluation of the model has been done based on errors $(IE_E, ISE_E, and\ IAE_E)$ and control efforts (CE). Furthermore, the suggested approach is also implemented for LFC of single-area power systems due to load disturbance. The results reveal the supremacy of the suggested technique, which provides good load disturbance attenuation and fast and smooth response. In the future, the designed method will be carried out with the comparative analysis with other advanced techniques such as: sliding mode control, H_∞ control, and Active Disturbance Rejection Control (ADRC). The designed PID based IMC scheme will be implemented in the Microgrid as an application.

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