Optimal Energy Flow of Unbalanced Power Distribution System Integrated with District Heating System

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***Abstract*—This paper presents the decentralized 15-minute time interval multi-period energy optimization of an active Unbalanced three-phase Power Distribution System (UPDS) interconnected with the District Heating System (DHS). The UPDS subsystem problem is formulated as a bi-level problem, where level-1 solves the linear three-phase power flow approximation, and level-2 solves the nonlinear convex versions of the subsystem problem. The variable mass flow rate in DHS brings increased system flexibility compared to the constant flow rate and, therefore, enhances energy optimization of the heating system. The McCormick envelopes based polyhedral relaxations and the Second Order Cone Programming (SOCP) relaxations are employed to convexify the nonconvexities involved in the integrated UPDS-DHS system. The relaxation error in each subsystem problem is iteratively reduced below predefined acceptance levels. The proposed approach minimizes the integrated system's multi-period operation cost and is implemented on an active unbalanced IEEE-13 bus distribution network and a 30-node DHS.**

***Index Terms*—ADMM, DHS, McCormick envelopes, Optimal power flow, SOCP, unbalanced power distribution system**

I. Introduction

With the advancement of technology and the living style of human society, energy consumption is increasing rapidly. Therefore finding a sustainable, efficient, and safe energy utilization model is becoming more important to cater to the increased energy consumption. The recent advances in renewable energy penetration and integrated energy systems, consequently reducing dependence on fossil fuel resources and increasing system flexibility, create new avenues for energy internet business models. The communication and methods of information exchange in the energy internet are overturning the traditional business models [1]. With the technological progress of the energy internet, traditional power distribution networks are beginning to interact with other energy systems, such as natural gas distribution systems and cooling and DHS systems. Along with the integrated energy systems, integrating renewable energy sources such as PV, wind, etc., brings additional challenges to the system's operation. Since the heating system operation is more complex and challenging to solve compared to the other energy systems, the research on combined heat-electricity, especially at distribution levels, is limited to the field of balanced power flow calculations. In reality, the power distribution systems are unbalanced; therefore, the heating system's operation interconnected with a UPDS is worth studying.

The integrated energy system provides extensive interaction among heating and electricity energy systems through coupling facilities such as combined heat and power (CHP) units, heat pumps, and water pumps [2]. The integrated energy system can improve overall flexibility compared to the traditional decoupled energy systems. Under the assumption of constant flow mode, the optimal dispatch can be modeled as a convex problem with significantly reduced computational complexities as presented in [3]. Compared to the constant flow models, variable mass flow rate in a heating system can significantly increase the flexibility and savings in the electricity consumption of water pumps [4]. Asl et al., in [5], focus on the operation of an integrated energy system comprising an unbalanced distribution system, natural gas, and heating system. However, the node method and constant mass flow rate are utilized to model the heating system to avoid computational complexities. Moreover, instead of a recent classical approach like SOCP, SDP, etc., the metaheuristic optimization solution technique is proposed as the solution strategy.

The information exchange methods and privacy issues have been studied very well in the developed decentralized operational solution strategies. Several solution algorithms are proposed to obtain the coordinated schedule of two subsystems solved alternatively. Apart from the heuristic algorithms [6], classical mathematical solution approaches including generalized benders decomposition [3], optimality condition decomposition [7], heterogeneous decomposition [8], the augmented Lagrangian method [9], and the alternating direction method of multipliers (ADMM) [10] are employed for efficient decentralized operation. In the works of literature above, each system solves its subproblem and exchanges only boundary information related to the coupling facilities to obtain the

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optimal solution while preserving privacy. This paper studies the ADMM-based decentralized multi-period energy operation of DHS integrated with an unbalanced distribution network while minimizing the relaxation errors in each subsystem. The UPDS subproblem is modeled as a bi-level programming problem, whereas the DHS subproblem is a variable temperature - variable mass flow rate based nonlinear programming problem. The McCormick envelopes and second-order cone programming (SOCP) are employed to solve the nonconvexities involved in the integrated system.

The rest of the article is organized as follows. The problem formulation of UPDS and DHS and their objective functions are described in Section II. The ADMM-based decentralized solution technique aiming to minimize system operation cost while reducing the relaxation error is introduced in Section III. The simulation studies and result discussions are shown in Section IV, and the conclusions are drawn in Section V.

II. Problem Formulation

A. Objective function of Integrated UPDS-DHS Energy System

The integrated UPDS-DHS energy system's objective is to minimise the entire operating costs of both the heating and electricity distribution system. The objective function is expressed as follows

$$OF = \sum_{t \in T} [Cost_t^{UPDS} + Cost_t^{DHS}] \tag{1}$$

Where, the overall cost of operation of an electrical system includes the cost of power purchased from the main grid and the cost of natural gas fired co-generating units (CHP). The operation cost of DHS is expressed as the cost of total heat consumption.

B. UPDS Modeling

The Branch Flow Model (BFM) based three-phase real and reactive power balance equations are expressed as [11]

$$P_{jk,t}^{aa} - \sum_{b \in \phi_k} l_{jk,t}^{ab} (r_{jk}^{ab} cos(\theta_{jk,t}^{ab} + x_{jk}^{ab} sin(\theta_{jk,t}^{ab})) = \sum_l P_{kl,t}^{aa} + p_{k,t}^{a,L} - (P_{k,t}^{a,g} + P_{k,t}^{a,chp} - P_{k,t}^{a,hp} - P_{k,t}^{a,wp} + P_{k,t}^{a,pv}) \tag{2}$$

$$Q_{jk,t}^{aa} - \sum_{b \in \phi_k} l_{jk,t}^{ab} (x_{jk}^{ab} cos(\theta_{jk,t}^{ab} - r_{jk}^{ab} sin(\theta_{jk,t}^{ab})) = \sum_l Q_{kl,t}^{aa} + q_{k,t}^{a,L} - (Q_{k,t}^{a,g} + Q_{k,t}^{a,chp} - Q_{k,t}^{a,wp} + Q_{k,t}^{a,pv}) \tag{3}$$

where, P_{jk}^{aa} and Q_{jk}^{aa} are real & reactive power flows in phase-a of line $(j,k) \in \mathcal{L}$, respectively. $r_{jk}^{ab} + jx_{jk}^{ab}$ is the mutual impedance of line (j,k) between phases a & b. $P_{k,t}^{a,g/chp}$ and $Q_{k,t}^{a,g/chp}$ are the real & reactive power generation in phase-a from grid/chp at bus k, time t. $\phi_k = \{a,b,c\}$ The voltage drop equation is expressed as

$$v_{k,t}^a - v_{j,t}^a = -\sum_{b \in \phi_k} 2\Re[S_{jk,t}^{ab} (z_{jk}^{ab})^*] + \sum_{b \in \phi_k} |z_{jk}^{ab}|^2 l_{jk,t}^{bb} + \sum_{b1,b2 \in \phi_k, b1 \neq b2} 2\Re\left[z_{jk}^{ab1} l_{jk,t}^{b1b2} \left(\angle(\theta_{jk,t}^{b1b2})\right) \left(z_{jk}^{ab2}\right)^* \right] \tag{4}$$

The relationship between line power flow, current and voltage is given as

$$(P_{jk,t}^{aa})^2 + (Q_{jk,t}^{aa})^2 = v_{j,t}^a l_{jk,t}^{aa}, \qquad (l_{jk,t}^{ab})^2 = l_{jk,t}^{aa} l_{jk,t}^{bb} \tag{5}$$

The SOCP relaxation of constraints (5) are expressed as

$$(P_{jk,t}^{aa})^2 + (Q_{jk,t}^{aa})^2 \leq v_{j,t}^a l_{jk,t}^{aa} \quad \& \quad (l_{jk,t}^{ab})^2 \leq l_{jk,t}^{aa} l_{jk,t}^{bb} \tag{6}$$

C. DHS Modeling

There are two main modeling parts of the DHS namely, hydraulic and thermal.

1) Hydraulic: The mass flow continuity and major pressure loss constraints are expressed as

$$\dot{w}_{m,t}^{in} - \dot{w}_{m,t}^{out} = \sum_n \dot{w}_{mn,t} - \sum_l \dot{w}_{lm,t} \quad \forall\, m \in \mathcal{M} \tag{7}$$

$$\begin{aligned} P_{m,t}^s - P_{n,t}^s &= \mu_{mn} \dot{w}_{mn,t}^2, \\ P_{n,t}^r - P_{m,t}^r &= \mu_{mn} \dot{w}_{mn,t}^2 \qquad \forall\, (m,n) \in \mathcal{L} \end{aligned} \tag{8}$$

where $P_{m,t}^s/P_{m,t}^r$ is the nodal pressure of supply/return network. μ_{mn} is the pressure loss coefficient. The electricity consumption of water pump is expressed as

$$P_{k,t}^{wp} = \frac{\dot{w}_{m,t}^{in}(P_{m,t}^s - P_{m,t}^r)}{\eta^{wp} \rho_w}, \; P_k^{wp} \leq \bar{P}_k^{wp} \quad \forall\, k \in \mathcal{K}^{wp} \tag{9}$$

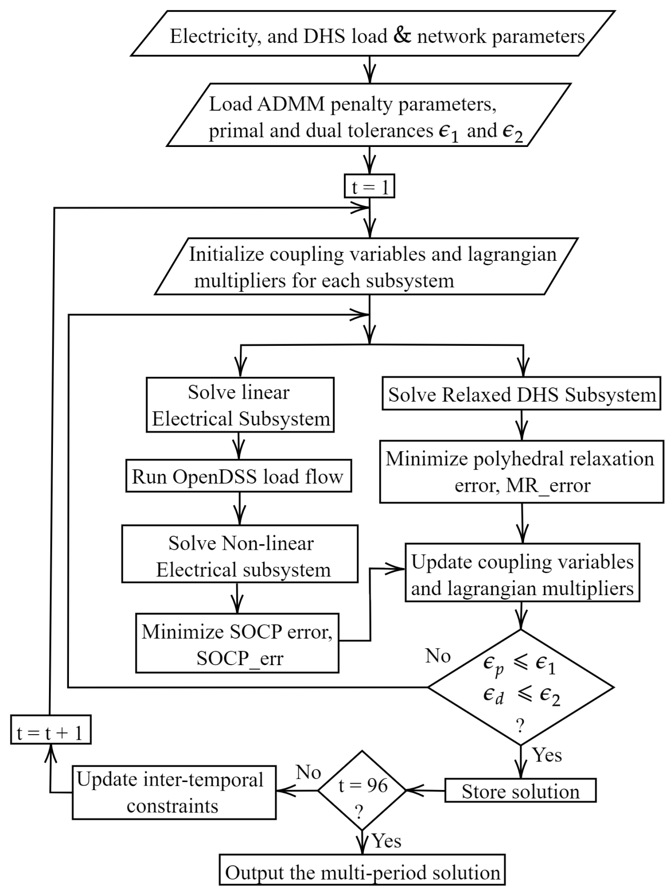

Fig. 1. Flowchart of Solution Strategy

2) Thermal: The heat energy produced by HP and CHP plants are expressed as

$$P^{chp}_{k,t}+H^{chp}_{m,t}=\eta_{chp}F^{chp}_{t},\ H^{chp}_{m,t}=\xi_{chp}F^{chp}_{t},\ H^{hp}_{m,t}=\eta_{hp}P^{hp}_{k,t} \tag{10}$$

The heat balance constraint are expressed as

$$H^{chp}_{m,t}+H^{hp}_{m,t}=c_w\dot{w}^{in}_{m,t}\Delta Ts_{m,t} \tag{11}$$

$$H^{d}_{m,t}=c_w\dot{w}^{out}_{m,t}\Delta Tr_{m,t} \tag{12}$$

The nodal temperature mix in both heat networks are formulated as

$$T^{s}_{m,t}=\frac{T^{s,in}_{m,t}\dot{w}^{in}_{m,t}+\sum_n T^{s,out}_{nm,t}\dot{w}_{nm,t}}{\dot{w}^{in}_{m,t}+\sum_n \dot{w}_{nm,t}}\quad \forall\ m,n\in\mathcal{M} \tag{13}$$

$$T^{r}_{n,t}=\frac{T^{r,out}_{n,t}\dot{w}^{out}_{n,t}+\sum_m T^{r,out}_{nm,t}\dot{w}_{nm,t}}{\dot{w}^{out}_{n,t}+\sum_m \dot{w}_{nm,t}}\quad \forall\ m,n\in\mathcal{M} \tag{14}$$

where $T^{s,out}_{nm,t}$ and $T^{r,out}_{nm,t}$ are the pipeline (n,m) outlet temperature in supply and return network, respectively.
The first order partial differential equation depicting water flow and temperature dynamics is expressed as

$$\frac{\partial T}{\partial t}+\frac{\dot{w}}{A\rho_w}\frac{\partial T}{\partial x}+\frac{\lambda}{A\rho_w c_w}(T-T^g)=0 \tag{15}$$

where λ is the thermal transfer coefficient.

III. Solution Procedure

This section presents an ADMM-based decentralized 15-minute time interval multi-period energy optimization of the integrated UPDS-DHS energy system. The UPDS problem is solved as a bi-level subproblem, where level-1 solves the linear UPDS subsystem to obtain the control settings of voltage control facilities and the power generation/consumption of coupling facilities. The information obtained from level-1 is utilized to approximate the line current angles computed using a three-phase OpenDSS load flow algorithm. The angles are therefore treated as constant values for the level-2 problem, which solves the convex nonlinear problem. The solution to the original nonconvex problem is retrieved in iterations from the convex relaxation as shown in the flowchart of solution strategy figure 1.

IV. Results and Discussion

The simulations are performed on a computing system with Intel(R) Core(TM) i5-7400 CPU @3GHz & 16 GB RAM installed. The proposed solution approach is coded in GAMS Distribution 37.1.0 integrated with MATLAB 2021b. The integrated UPDS-DHS multi-period optimization problem is solved using a BARON solver with GAMS distribution.

The topology of the integrated UPDS-DHS system shows the location of the coupling facilities and photovoltaic arrays, as shown in Fig. 3. The unbalanced IEEE 13-bus distribution system is suitable for the convergence test of a solution approach for a very unbalanced distribution network. The 30-node DHS, with a total length of 6.6km, contains 17

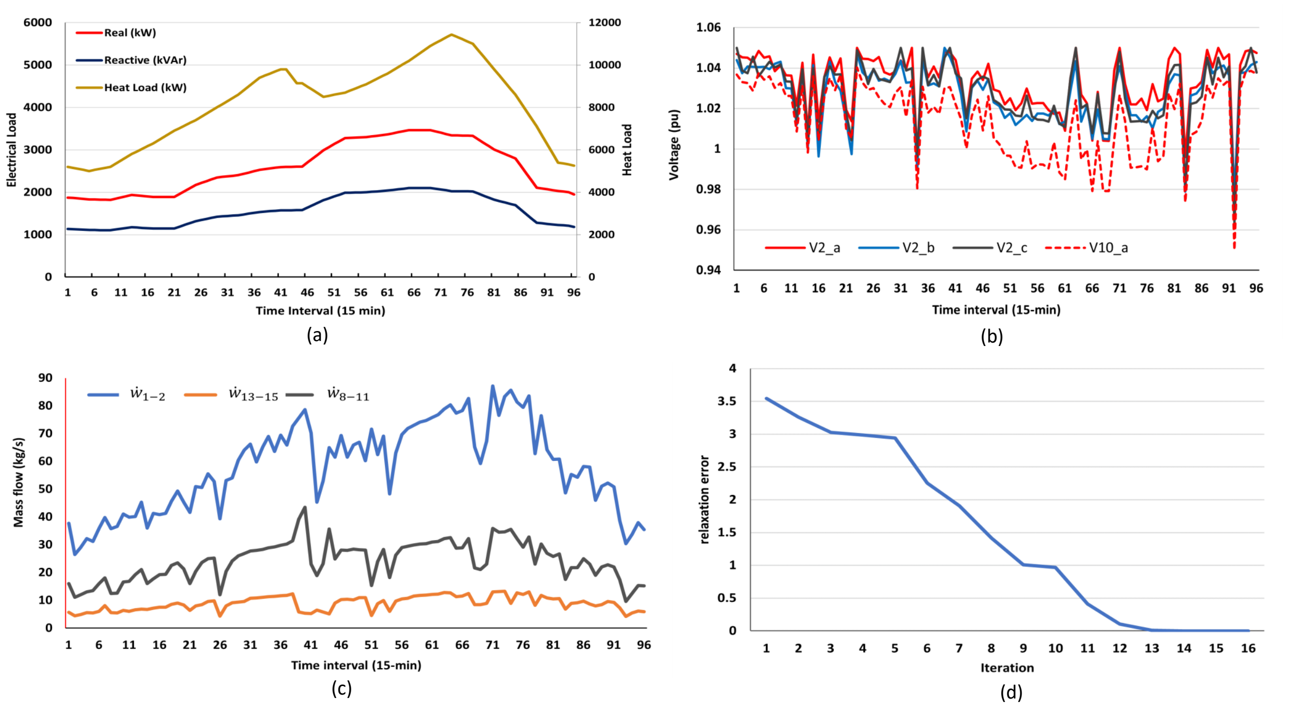

Fig. 2. (a) Load demand. (b) Node voltages. (c) Pipeline mass flow rate. (d) Evolution of relaxation error.

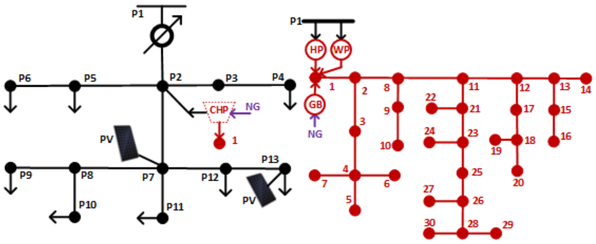

Fig. 3. Illustration of UPDS-DHS System

heat exchangers and 29 pipelines [12]. The 15-minute time interval UPDS and DHS load demand curve are shown in Fig. 2(a). The size of the solar panel is considered 50% of the rated load demand located at the respective phase of the nodes. And the size of the PV inverter is set by taking into account the power factor of 0.9 [11]. Fig. 2(b) shows the 15-minute time step 24-hours phase voltage of node $P2$ and $P10$. Node $P2$ is a three-phase bus located near the substation, whereas node $P10$ is a single-phase leaf node. It can be observed that the phase voltages are within the prescribed voltage limits of $\pm5\%$. The mass flow rate in DHS for pipeline $(1,2)$, $(8,11)$ and $(13,15)$ are shown in Fig. 2(c). The variable mass flow rate allows more flexibility in the system and makes most of the heat inertia compared to the predefined hydraulic conditions with a constant flow model. The solution obtained from the relaxed subproblems of each subsystem may not be feasible for their original nonconvex counterparts. Therefore, recovering a feasible solution is imperative to get a meaningful solution. Fig. 2(d) shows the evolution of the relaxation error in iterations at a specific time interval. The relaxation error is minimized iteratively until a predefined tolerance level is reached.

V. Conclusion

This paper proposes a day-ahead ADMM-based decentralized energy operation of an unbalanced distribution network interconnected with a district heating system. The solution of the original nonconvex subproblem for each subsystem is iteratively retrieved from their relaxed counterparts. The relaxation error is below the predefined acceptance levels in each time interval for each subsystem. The results analysis demonstrates the effectiveness of the proposed solution approach, where phase voltages are well within limits in the UPDS subproblem, and variables flow rate makes most of the heat inertia in the DHS subproblem. In future studies, it will be interesting to study the intensive integration of multi-energy systems into the unbalanced power distribution network.

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