2022 IEEE Global Conference on Computing, Power and Communication Technologies (GlobConPT).
India Habitat Centre, Lodhi Road, New Delhi, India. Sep 23-25, 2022

ADMM-based Optimal Energy Flow of Unbalance Distribution Networks Integrated with Natural Gas Distribution Systems

Abhimanyu Sharma, SIEEE
Department of Electrical Engineering
Indian Institute of Technology Roorkee
Roorkee, India - 247667
Email: abhimanyu1@ee.iitr.ac.in

Narayana Prasad Padhy, SMIEEE
Department of Electrical Engineering
Indian Institute of Technology Roorkee
Roorkee, India - 247667
Email: nppadhy@ee.iitr.ac.in

2022 IEEE Global Conference on Computing, Power and Communication Technologies (GlobConPT) | 978-1-6654-9365-9/22/$31.00 ©2022 IEEE | DOI: 10.1109/GLOBCONPT57482.2022.9938212

***Abstract*—This paper studies the complex energy coupling relationship of unbalanced power distribution network integrated with the natural gas distribution system. An Alternating Direction Method of Multipliers (ADMM) based decentralized solution approach is proposed to achieve the multi-period synergistic operation of the power distribution and natural gas distribution networks. A bi-level solution strategy is used to solve the power distribution subsystem problem, where level-1 is the linearized version of three-phase optimal load flow and level-2 solves the convex nonlinear problem. The level-1 problem helps to determine the status of voltage control devices whereas the level-2 problem obtains the optimal and feasible solution of the optimal load flow problem. The non-convexities present in the integrated energy system are dealt with the Second Order Cone Programming (SOCP) and Quadratic Relaxation (QR). The purposed solution procedure is validated using IEEE 13-bus distribution system integrated with 6-bus natural gas network. The numerical results corroborate the applicability of the proposed approach to minimize the operation cost of the integrated energy system.**

***Index Terms*—Unbalanced Distribution System, Natural Gas, multi-period Optimal Power Flow, ADMM, SOCP**

I. Introduction

With the increasing pressure from environmental protection and efficient utilization of energy, the tremendous penetration of distributed generation and the integrated energy technologies has been rapidly developed, such as Combined Heat and Power Dispatch (CHPD), natural gas power generation, renewable energy heating. Based on the energy coupling relationship, the integrated energy supply system has been witnessing the transition from independent design, planning and operation to joint system design, planning and operation [1]. Due to the low-cost and high-efficiency of the natural gas (NG), gas fired distributed generating units in electricity grid are rapidly becoming optimal choice for the integration of new generating units. Moreover, NG has already become the second largest source of world energy consumption [2]. Some integrated energy models have been suggested to jointly operate and analyse the electricity and natural gas systems. In the pioneering work of [3], the Optimal Power Flow (OPF) model for gas integrated electricity networks was proposed to achieve optimal and feasible solution for both systems. The proposed model was then extended in [4] to consider the multi-period joint operation of gas integrated electricity systems. However, the intensified integration and interdependency of electricity and NG networks have been widely studied on the transmission level [2]–[7].

The aforementioned efforts have made a significant contribution to the integration of multi-energy systems and promoted the synergistic operation of the electricity and gas networks. However, these works are either based on the assumption of the single vertically integrated utility or operated on the transmission levels as compared to the distribution levels. Moreover, there are significant modeling differences in the transmission and distribution operating levels in the integrated energy systems. Firstly, the distribution system's network structure, parameters, and power flow algorithms are different from that of the transmission network. Secondly, distribution networks are mostly unbalanced compared to the transmission network. Third, the huge amount of NG and long-distance pipes in the transmission network cause substantial pressure loss, necessitating the use of compressors to compensate [8], but pressure loss at the distribution level is significantly smaller and does not necessitate the use of compressors [9]. In case of a single vertically operated utility, a single centralised decision maker manages energy flow optimization, which may result into increased computational burden and privacy issues. Whereas in reality, the power and gas networks are managed by independent entities, therefore, a decentralized computational framework is imperative to optimize the integrated energy system with minimal privacy concerns.

Over the past few years, various distributed and decentralized solution approaches are applied to power system operation problems, which include lagrangian relaxation (LR) [10], [11], dynamic multiplier based LR [12] and auxiliary problem principle (APP) [13], [14]. Recently, ADMM [15] has gained more attention due to its computational efficiency, decomposable nature and faster convergence as compared to the aforementioned traditional distributed solution approach. Hence, ADMM is widely studied and applied to various power system operation and planning problems. A detailed literature survey on applications of ADMM pertaining to smart power grids is conducted in [1]. An ADMM based decentralized computational framework is proposed to optimize the operation cost of unbalanced distribution network integrated with NG system. Whereas the above literature ignored the integration of energy systems with unbalanced distribution systems and, therefore, fails to address the computational complexities involved. Further, the exact solution to the original non-convex problem is recovered iteratively by reducing the relaxation error.

To overcome the above research gaps, the specific contributions are listed below:

1) An ADMM based decentralized computational solution strategy with minimal privacy concern is proposed for the multi-period synergistic operation of unbalanced distribution systems and natural gas distribution networks.
2) A bi-level optimization solution approach for electrical subsystem with coupling devices from natural gas subsystem is suggested, where level-1 optimize the control of voltage control devices using a linearized 3-phase optimal power flow. With the information obtained about the coupling devices and voltage control devices from level-1, the line current angles are obtained using the OpenDSS load flow. The level-2 nonlinear programming problem obtains the feasible and optimal solution by solving the approximate three-phase nonlinear convex optimal load flow model.
3) The exact solution to the original multi-period optimization problem is recovered by iteratively reducing the relaxation error.

The rest of the article is organized as follows: Section II shows the problem formulation with integrated energy system objective functions and detailed modeling of the energy systems, the ADMM based solution procedure is shown in Section III, at last; section IV and V shows the results and conclusion portions, respectively.

II. Problem Formulation

A. Objective function of Integrated Energy System

The integrated energy system's objective is to minimise the entire operating costs of both the electricity and natural gas networks. The objective function is expressed as follows

$$OF = \sum_{t \in T} [Cost_t^{elect} + Cost_t^{NG}] \quad (1)$$

Where, the overall cost of operation of an electrical system includes the cost of power purchased from the main grid and the cost of natural gas fired generating units (NGUs). The operation cost of natural gas system is expressed as the cost of total gas supplied.

$$Cost^{elect} = \sum_{t \in T} [Cost_t^{grid} + Cost_t^{NGUs}] \quad (2)$$

$$Cost^{NG} = \sum_{t \in T} \alpha_t G_{s,t} \quad (3)$$

The $Cost_t^{grid}$ and $Cost_t^{NGUs}$ are expressed as quadratic cost functions

$$Cost_t^{grid} = \sum_{i \in M} \sum_{a \in \phi} (a_0 (P_{i,t}^{a,g})^2 + a_1 P_{i,t}^{a,g} + a_2)$$

$$Cost_t^{NGUs} = \sum_{j \in N_{dg}} \sum_{a \in \phi} (b_0 (P_{j,t}^{a,dg})^2 + b_1 P_{j,t}^{a,dg} + b_2)$$

B. Modeling of Unbalanced Eletrical Distribution Network

The nonlinearity in three-phase power flow equations due to mutual coupling in 3-phases are approximated. The phase-decoupled formulation is obtained by decoupling the voltage and power balance equations on a perphase basis. This approximation is based on the following two assumptions (see [16], [17] for more details)

1) Assumption 1: The phase voltage unbalance rate is not large. This condition applies to a distribution system that meets the ANSI requirements for phase unbalance and bus voltages [18].
$$\frac{V_i^a}{V_i^b} \simeq \frac{V_i^b}{V_i^c} \simeq \frac{V_i^c}{V_i^a} = e^{j2\pi/3}$$
2) Assumption 2: The angle difference between phase currents are approximated. For a certain power line $(i,j) \in \mathcal{L}$, the ampere flow for phase a and b are $I_{ij}^a = |I_{ij}^a| \angle \theta_{ij}^a$, and $I_{ij}^b = |I_{ij}^b| \angle \theta_{ij}^b$. The phase currents angle difference is given as $\theta_{ij}^{ab} = \angle \theta_{ij}^a - \angle \theta_{ij}^b$. These angles are calculated using equivalent OpenDSS load flow solution with the information obtained from the linearized version (level-1) of electrical system. Thereafter, these angles are treated as constants for the nonlinear version (level-2).

Using the above approximations, the Branch Flow Model (BFM) based power flow equations are expressed as [16], [17]

$$P_{ij,t}^{aa} - \sum_{b \in \phi_j} l_{ij,t}^{ab} (r_{ij}^{ab} cos(\theta_{ij,t}^{ab} + x_{ij}^{ab} sin(\theta_{ij,t}^{ab})) = \sum_k P_{jk,t}^{aa} + p_{j,t}^{a,L} - (P_{j,t}^{a,g} + P_{j,t}^{a,dg} + P_{j,t}^{a,chp}) \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (4)$$

$$Q_{ij,t}^{aa} - \sum_{b \in \phi_j} l_{ij,t}^{ab} (x_{ij}^{ab} cos(\theta_{ij,t}^{ab} - r_{ij}^{ab} sin(\theta_{ij,t}^{ab})) = \sum_k Q_{jk,t}^{aa} + q_{j,t}^{a,L} - (Q_{j,t}^{a,g} + Q_{j,t}^{a,dg} + Q_{j,t}^{a,chp}) \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (5)$$

where, P_{ij}^{aa} and Q_{ij}^{aa} are real & reactive power flows in phase-a of line $(i,j) \in \mathcal{L}$ at time t, respectively. $r_{ij}^{ab} + jx_{ij}^{ab}$ is the mutual impedance of line (i,j) between phases a & b. $P_{j,t}^{a,g/dg/chp}$ and $Q_{j,t}^{a,g/dg/chp}$ are the real & reactive power generation in phase-a from grid/NGUs/chp at bus j, time t. $\phi = \{a,b,c\}$

The voltage drop equation for line (i,j) is expressed as

$$v_{j,t}^a - v_{i,t}^a = -\sum_{b \in \phi_j} 2\,\mathrm{Re}\{[S_{ij,t}^{ab} (z_{ij}^{ab})^*]\} + \sum_{b \in \phi_j} |z_{ij}^{ab}|^2 l_{ij,t}^{bb} + \sum_{b1,b2 \in \phi_j, b1 \neq b2} 2\,\mathrm{Re}\Big\{\Big[z_{ij}^{ab1} l_{ij,t}^{b1b2} \left(\angle(\theta_{ij,t}^{b1b2})\right) (z_{ij}^{ab2})^*\Big]\Big\} \quad (6)$$

The relationship between line power flow, current and voltage is given as

$$(P_{ij,t}^{aa})^2 + (Q_{ij,t}^{aa})^2 = v_{i,t}^{a} l_{ij,t}^{aa} \tag{7}$$

$$(l_{ij,t}^{ab})^2 = l_{ij,t}^{aa} l_{ij,t}^{bb} \tag{8}$$

Equation (8) related the current variables. The non-convexities in constraints (7) $\sim$ (8) are relaxed using SOCP relaxation as shown below

$$(P_{ij,t}^{aa})^2 + (Q_{ij,t}^{aa})^2 \leq v_{i,t}^{a} l_{ij,t}^{aa} \quad \& \quad (l_{ij,t}^{ab})^2 \leq l_{ij,t}^{aa} l_{ij,t}^{bb}$$

Linear approximation of Three-phase load flow: As compared to the line power flow, the line losses are quite smaller and therefore, with this assumption, the linear approximation ignores the effect of power loss on load flow equations [19]. The linear power flow approximation can be expresses as

$$P_{ij,t}^{aa} = \sum_{k} P_{jk,t}^{aa} + p_{j,t}^{a,L} - (P_{j,t}^{a,g} + P_{j,t}^{a,dg} + P_{j,t}^{a,chp}) \tag{9}$$

$$Q_{ij,t}^{aa} = \sum_{k} Q_{jk,t}^{aa} + q_{j,t}^{a,L} - (Q_{j,t}^{a,g} + Q_{j,t}^{a,dg} + Q_{j,t}^{a,chp}) \tag{10}$$

$$v_{j,t}^{a} - v_{i,t}^{a} = -\sum_{b \in \phi_j} 2\,\mathrm{Re}\big\{[S_{ij,t}^{ab}(z_{ij}^{ab})^*]\big\} \tag{11}$$

Although the linearized power flow doesn't include voltage drop due to power loss, however, it is significantly accurate in representing bus voltages [19].

C. Modeling of Natural Gas Distribution System

For the modeling of natural gas distribution network, it is assumed that the gas directions are predefined and fixed [20]. Moreover, the compressors are not required for distribution networks due to much lower pressure loss compared to the transmission levels [9]. The Weymouth equation describe the relationship between the NG flow and pressure loss in the pipelines expressed as in (12). Equations (13) $\sim$ (14) represent the physical characteristics of the pipeline. The gas flow balance equation is shown as in (15).

$$G_{mn,t}^2 = C_{mn}(pr_{m,t}^2 - pr_{n,t}^2) \tag{12}$$

$$G_{mn}^{min} \leq G_{mn,t}^2 \leq G_{mn}^{max} \tag{13}$$

$$pr_{m}^{min} \leq pr_{m}^2 \leq pr_{m}^{max} \tag{14}$$

$$\sum_{mn \in \mathcal{P}} G_{mn,t} + G_{m,t}^{S} - G_{m,t}^{L} - G_{m,t}^{dg} = 0 \tag{15}$$

Where, $G_{mn,t}$ is the gas flow in pipeline $(m,n) \in \mathcal{P}$. C_{mn} is the pipeline parameter. G_m^S, G_m^L and pr_m are the nodal gas supply, load demand and pressure, respectively. Square of pressure variable is itself considered as a variable to avoid square term. Quadratic equality constraints of form $f = aG^2$ as in LHS of (12) are reformulated using Second Order Cone constraints. Further, Linear inequality constraints are included to cut off infeasible solutions.

$$\mathbf{QR}(f = aG^2) \triangleq \left\{ \begin{array}{c} f \geq aG^2, \\ f \leq a\left[(\overline{G} + \underline{G})G - \underline{G}\overline{G}\right] \end{array} \right. \tag{16}$$

As the unbalanced distribution system and gas distribution network are interconnected by the natural gas fired generating units (NGUs), the following coupling constraint should be satisfied

$$G_{m,t}^{dg} \geq (1/\mu)(\gamma_1 (p_{j,t}^{dg})^2 + \gamma_2 p_{j,t}^{dg} + \gamma_3) \tag{17}$$

$$p_{j,t}^{dg} = \sum_{a \in \phi_j} p_{j,t}^{a,dg} \tag{18}$$

where $\gamma_1(MBtu/MWh^2)$, $\gamma_2(MBtu/MWh)$ and $\gamma_3(MBtu)$ are heat curve coefficients. μ is MBtu to kcf conversion factor. Nodes $j \in \mathcal{N}_{dg}$ and $m \in \mathcal{I}_{dg}$ are the coupling nodes of electrical and gas distribution networks, respectively.

Algorithm 1 ADMM based Integrated Energy System

1: Set the initial values for coupling variable z_0, lagrangian multipliers ς_0 and ϑ_0. Set ADMM residuals tolerances ϵ_1 and ϵ_2; set the iteration index $k = 0$

2: Solve (19) and (20)

$$x_{k+1} = \arg\min_{x \in \Omega_x} (L(x, z_k, \varsigma_k)) \tag{19}$$

$$y_{k+1} = \arg\min_{y \in \Omega_y} (L(y, z_k, \vartheta_k)) \tag{20}$$

3: Update the coupling variable according to (21)

$$z_{k+1} = (x_{k+1} + y_{k+1})/2 \tag{21}$$

4: Update the Lagrangian multipliers (22) & (23)

$$\varsigma_{k+1} = \varsigma_k + \rho(x_{k+1} - z_{k+1}) \tag{22}$$

$$\vartheta_{k+1} = \vartheta_k + \rho(y_{k+1} - z_{k+1}) \tag{23}$$

5: If $\epsilon^{primal} \leq \epsilon_1$ and $\epsilon^{dual} \leq \epsilon_2$ is satisfied, then exit; else set $k = k + 1$ and go to step 2. Where

$$\epsilon_{k+1}^{primal,1} = \|\varsigma_{k+1} - \varsigma_k\|_2^2 \tag{24}$$

$$\epsilon_{k+1}^{primal,2} = \|\vartheta_{k+1} - \vartheta_k\|_2^2 \tag{25}$$

$$\epsilon_{k+1}^{dual} = \rho(\|z_{k+1} - z_k\|_2^2) \tag{26}$$

III. Solution Procedure

In this section, an ADMM based decentralized computational framework is proposed to solve the integrated energy system optimization problem iteratively. The level-1 linearized version of electrical distribution subsystem is solved to obtain the ON/OFF status of the capacitor banks, tap position of OLTCs, and the power output of NGUs. The line current angles are approximated using load flow algorithm utlizing the output of the level-1 problem as input to it. The angles obtained from the power flow solution are treated as constant values for the level-2 problem. The solution to the original problem is recovered iteratively from the convex relaxation as shown in flowchart of solution strategy figure 1. The objective function (1) is decomposed into two convex functions $f(x)$ and $g(y)$. The convex feasible regions of the electrical and

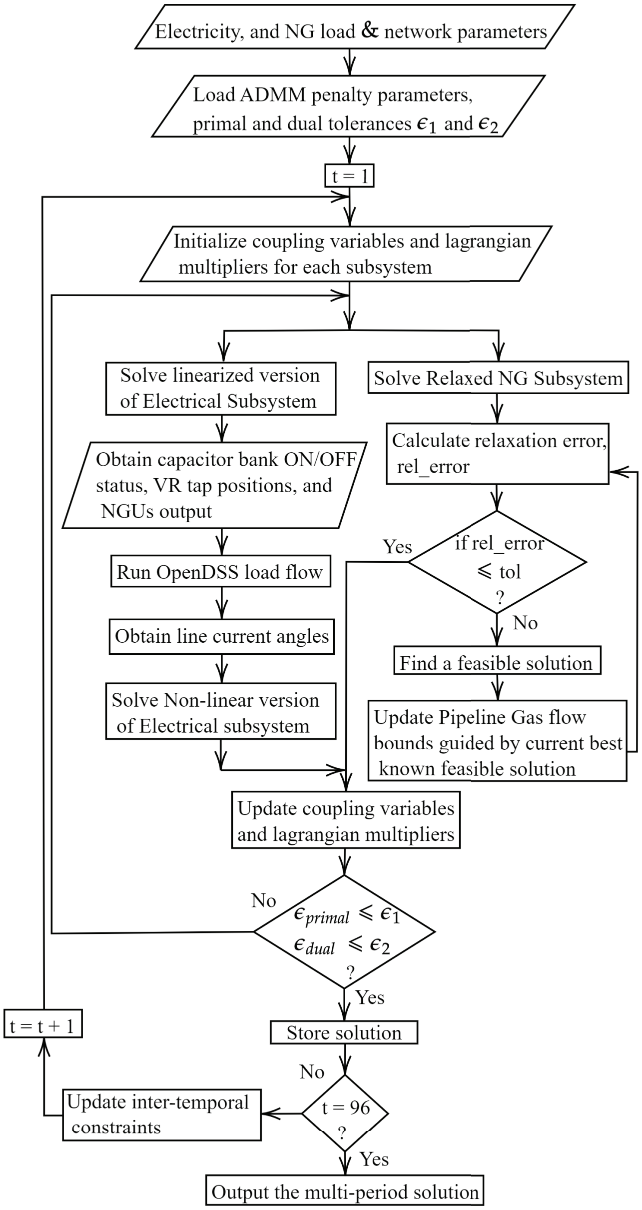

Fig. 1. Flow Chart of Solution Strategy

gas distribution subproblems are represented by Ω_x and Ω_y, respectively. The augmented lagrangian objective functions of the electrical and gas distribution subproblems are expressed as

$$L(x, z, \varsigma) = f(x) + \overrightarrow{\varsigma}^T (x - z) + \frac{\rho}{2} \|x - z\|^2 \tag{27}$$

$$L(y, z, \vartheta) = g(y) + \overrightarrow{\vartheta}^T (y - z) + \frac{\rho}{2} \|y - z\|^2 \tag{28}$$

where $x \in \Omega_x$ and $y \in \Omega_y$ are the decision variables of the two subproblems; z is the coupling variable. ς & ϑ are the dual variable and ρ is a constant parameter. The iterative steps of ADMM based decentralized solution procedure are given in Algorithm 1.

The detailed solution process with iterative procedure of solution recovery and ADMM is shown in figure 1.

IV. Results and Discussion

In this section, we use IEEE-13 bus and 6-node gas system to demonstrate the efficacy of the proposed decentralized solution strategy. The experiments are conducted on a computer system with Intel(R) Core(TM) i5-7400 CPU @3GHz & 16 GB RAM installed. The proposed algorithm is coded in GAMS Distribution 37.1.0 integrated with MATLAB 2019a. The integrated energy system optimization problem is solved using BARON solver in GAMS.

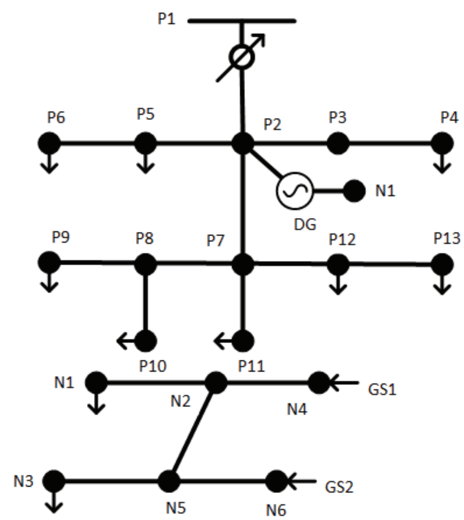

Fig. 2. Topology of Power13Gas6 System

A. IEEE 13-Bus Power distribution system with 6-Node Gas System

Figure 2 depicts the topology of the interconnected power distribution system and gas network. The IEEE 13-bus system [21] operating at 4.16 kV provide a good test of convergence of a program for a very unbalanced distribution system. The parameters of 6-node gas system are shown in Table I-III [22]. Table I contains the data related to gas pipeline parameters which are the gas flow boundaries and Weymouth equation parameter. The parameters in Table II are the gas suppliers boundary limits and their respective gas production cost multipliers. The cost coefficients of gas fired generating units (NGUs) and non-gas fired generating unit i.e. substation

TABLE I
Parameters of Gas Pipeline

From node (m)	To node (n)	C_{mn} ($kcf/psig$)	G_{mn}^{min} (kcf)	G_{mn}^{max} (kcf)
1	2	50.6	3200	5000
2	4	50.1	800	3000
2	5	37.5	1800	2650
3	5	43.5	1535	2500
5	6	45.3	2000	5800

TABLE II
PARAMETERS OF NATURAL GAS SUPPLIERS

Gas Supplier	Node (m)	Min Output	Max Output	α ($\$/kcf$)
GS1	4	1500	5000	7.1
GS2	6	2000	6000	7

power unit with their power production boundary limits are shown in Table III. The 15-min time interval daily load profiles of electrical and NG system are shown in figure 3.

TABLE III
COST COEFFICIENTS OF GENERATING UNITS

Unit	a_0	a_1	a_2	Pg^{min} (kW)	Pg^{max} (kW)
NGU	0.0016	52.04	587.8	300	1200
non-NGU	0.0015	53.1	627.23	600	3000

The energy optimization results are obtained by simulating the model in 15-min time interval for a single day. The outcomes achieved for each phase of unbalanced power distribution system with upper & lower loading conditions are tabulated in Table IV. The status of a 3-phase and a 1-phase capacitor bank with tap positions of a 32-step 3-phase voltage regulator are shown in this table. The substation power injection and system voltages of OPF solution obtained from GAMS are validated using OpenDSS load flow solution with same control settings. The average phase voltages for each phase in both loading conditions is well within the prescribed voltage boundaries of $\pm 5\%$.

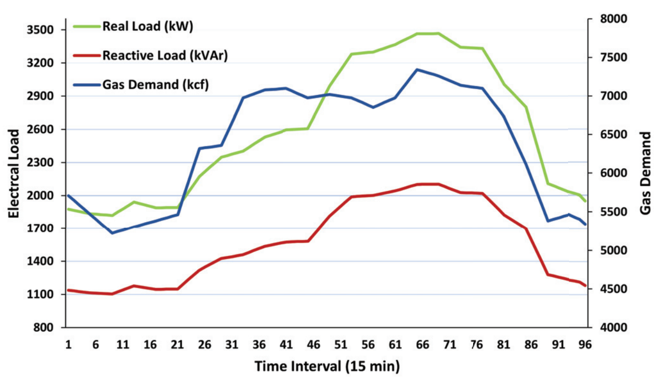

Fig. 3. One Day Load Profile

The phase voltage unbalance rate is under 3% for both upper & lower loading conditions as shown in figure 4. It can observed that the phase voltage unbalance is around $2.5-3\%$ for leaf nodes in both type of loadings. The more NGU penetration near the leaf nodes, the less will be the phase voltage unbalance. The phase voltage unbalance increases with higher unbalance loadings. The large amount of unbalance loading can also be observed from table IV. The three-phase voltages of DG node (node-2) and a leaf node (node-13) for a day are plotted in figure 5. The voltages of each phase in each time interval is within the desired voltage band i.e. $0.95-1.05(pu)$.

TABLE IV
ENERGY OPTIMIZATION RESULTS FOR POWER DISTRIBUTION SYSTEM

IEEE-13	Maximum Load			Minimum Load		
Optimal Load Flow settings from GAMS						
Phase	a	b	c	a	b	c
Regulator Tap	-4	-9	6	16	16	16
Capacitor1 Status	OFF	OFF	OFF	OFF	OFF	OFF
Capacitor2 Status	-	-	OFF	-	-	OFF
Subst.Power (MW)	1.218	0.84	1.237	0.569	0.385	0.594
Minimum Volt.(pu)	0.95	0.956	0.95	0.954	0.95	0.954
Maximum Volt. (pu)	1.049	1.05	1.05	1.05	1.045	1.05
Avg. Volt. (pu)	0.982	0.985	0.99	1.02	1.032	1.006
DG Power (kW)	100.01	100.01	100.01	100.23	100.23	100.23
Voltages and Substation Load Flow Validation Using OpenDSS						
Subst.Power (MW)	1.219	0.843	1.24	0.571	0.388	0.598
Minimum Volt. (pu)	0.95	0.956	0.95	0.954	0.95	0.952
Maximum Volt. (pu)	1.049	1.05	1.05	1.05	1.044	1.05
Avg. Volt. (pu)	0.982	0.985	0.99	1.02	1.03	1.004

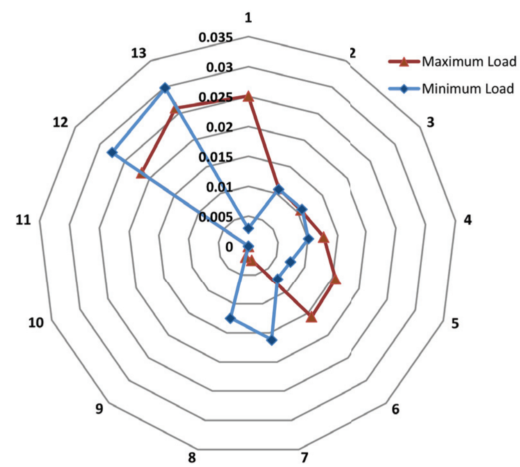

Fig. 4. Voltage Unbalance

The gas production cost for a day of each gas supplier is plotted in figure 6. The gas production of GS2 is higher than that of the GS1 due to its lower production cost. Albeit GS2 has lower production cost, the $(N5N2)$ pipeline gas flow and loading conditions of N1 & N3 restrict the gas production of GS2 to a amount below it's full capacity.

The primal variable error convergence characteristics of decentralized ADMM algorithm for random time intervals are shown in figure 7. It takes around 5-8 iterations to satisfy the primal variable convergence criteria for all time intervals.

V. CONCLUSION

This paper proposes a day-ahead decentralized computational framework at distribution level and develop synergistic operation between unbalanced power distribution system and gas distribution system. The solution to the original nonconvex problem of gas distribution system is successfully recovered from the relaxed problem using an iterative solution recovery process. The results analysis demonstrates how the operation of one energy system may impact the other. It is interesting

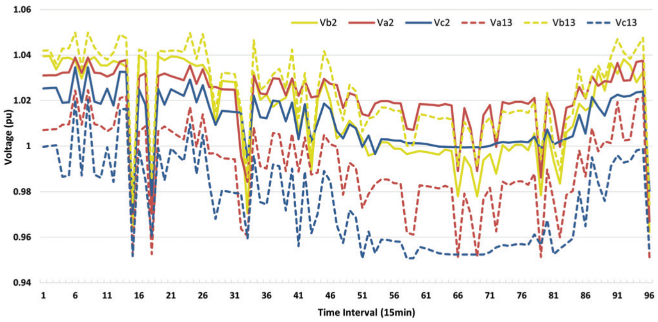

Fig. 5. Phase Voltages

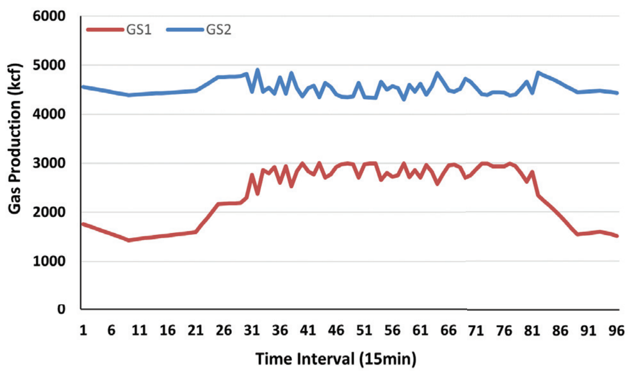

Fig. 6. Gas Production

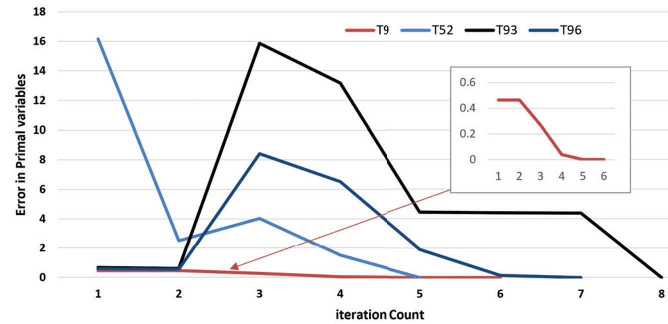

Fig. 7. Convergence curve for Primal Error

to study the operation flexibility brought by multi-energy systems at distribution levels including the impact of NGUs penetration on the phase voltage unbalance rate. The more interaction among energy systems at distribution level and interdependence in loads similar to energy hub concept will be interesting to analyze in future studies.

REFERENCES

[1] A. Maneesha and K. S. Swarup, "A survey on applications of alternating direction method of multipliers in smart power grids," *Renewable and Sustainable Energy Reviews*, vol. 152, p. 111687, 2021.

[2] Y. He, M. Yan, M. Shahidehpour, Z. Li, C. Guo, L. Wu, and Y. Ding, "Decentralized optimization of multi-area electricity-natural gas flows based on cone reformulation," *IEEE Transactions on Power Systems*, vol. 33, no. 4, pp. 4531–4542, 2018.

[3] S. An, Q. Li, and T. W. Gedra, "Natural gas and electricity optimal power flow," *2003 IEEE PES Transmission and Distribution Conference and Exposition (IEEE Cat. No.03CH37495)*, vol. 1, pp. 138–143 Vol.1, 2003.

[4] M. Chaudry, N. Jenkins, and G. Strbac, "Multi-time period combined gas and electricity network optimization," *Electric Power Systems Research - ELEC POWER SYST RES*, vol. 78, pp. 1265–1279, 07 2008.

[5] X. Zhang, M. Shahidehpour, A. Alabdulwahab, and A. Abusorrah, "Hourly electricity demand response in the stochastic day-ahead scheduling of coordinated electricity and natural gas networks," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 592–601, 2016.

[6] Y. Wen, X. Qu, W. Li, X. Liu, and X. Ye, "Synergistic operation of electricity and natural gas networks via admm," *IEEE Transactions on Smart Grid*, vol. 9, no. 5, pp. 4555–4565, 2018.

[7] Z. Zhang, C. Wang, S. Chen, Y. Zhao, X. Dong, and X. Han, "Multi-time scale co-optimized dispatch for integrated electricity and natural gas system considering bidirectional interactions and renewable uncertainties," *IEEE Transactions on Industry Applications*, pp. 1–1, 2022.

[8] X. Zhang, L. Che, M. Shahidehpour, A. Alabdulwahab, and A. Abusorrah, "Electricity-natural gas operation planning with hourly demand response for deployment of flexible ramp," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 3, pp. 996–1004, 2016.

[9] J. Chen, W. Zhang, Y. Zhang, and G. Bao, "Day-ahead scheduling of distribution level integrated electricity and natural gas system based on fast-admm with restart algorithm," *IEEE Access*, vol. 6, pp. 17557–17569, 2018.

[10] A. Conejo and J. Aguado, "Multi-area coordinated decentralized dc optimal power flow," *IEEE Transactions on Power Systems*, vol. 13, no. 4, pp. 1272–1278, 1998.

[11] A. Bakirtzis and P. Biskas, "A decentralized solution to the dc-opf of interconnected power systems," *IEEE Transactions on Power Systems*, vol. 18, no. 3, pp. 1007–1013, 2003.

[12] X. Lai, L. Xie, Q. Xia, H. Zhong, and C. Kang, "Decentralized multi-area economic dispatch via dynamic multiplier-based lagrangian relaxation," *IEEE Transactions on Power Systems*, vol. 30, no. 6, pp. 3225–3233, 2015.

[13] G. Cohen, "Auxiliary problem principle and decomposition of optimization problems," *Journal of Optimization Theory and Applications*, vol. 32, no. 3, p. 277–305, 1980.

[14] R. Baldick, B. Kim, C. Chase, and Y. Luo, "A fast distributed implementation of optimal power flow," *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 858–864, 1999.

[15] S. P. Boyd, N. Parikh, E. K. wah Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Found. Trends Mach. Learn.*, vol. 3, pp. 1–122, 2011.

[16] R. R. Jha, A. Dubey, C.-C. Liu, and K. P. Schneider, "Bi-level volt-var optimization to coordinate smart inverters with voltage control devices," *IEEE Transactions on Power Systems*, vol. 34, no. 3, pp. 1801–1813, 2019.

[17] S. Paul and N. P. Padhy, "A new real time energy efficient management of radial unbalance distribution networks through integration of load shedding and cvr," *IEEE Transactions on Power Delivery*, pp. 1–1, 2021.

[18] A. N. S. I. C84.1-2011, "American national standard for electric power systems and equipment—voltage ratings (60 hertz)," 2011.

[19] L. Gan and S. H. Low, "Convex relaxations and linear approximation for optimal power flow in multiphase radial networks," in *2014 Power Systems Computation Conference*, pp. 1–9, 2014.

[20] N. Keyaerts, "Gas balancing and line-pack flexibility." Available: https://www.mech.kuleuven.be/en/tme/research/energy_environment/Pdf/wpen2012-11.pdf, PhD Dissertation, September 2012. Accessed: 2022-02-20.

[21] K. P. Schneider, B. A. Mather, B. C. Pal, C.-W. Ten, G. J. Shirek, H. Zhu, J. C. Fuller, J. L. R. Pereira, L. F. Ochoa, L. R. de Araujo, R. C. Dugan, S. Matthias, S. Paudyal, T. E. McDermott, and W. Kersting, "Analytic considerations and design basis for the ieee distribution test feeders," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 3181–3188, 2018.

[22] Test data for natural gas. [ONLINE]. Available: https://sites.google.com/site/chengwang0617/home/data-sheet, 2016. Accessed: 2019-10-17.