Analytical Design of IMC-Based PID Controller for Non-minimum Phase Process with Time Delay

Vivek Kumar, R. Ranganayakulu, and G. Uday Bhaskar Babu

Abstract In this article, an internal model control (IMC) scheme-based proportional integral derivative (PID) controller is designed for controlling second order non-minimum phase system with time delay. The design uses IMC filters of higher order for realizing the controller. The novelty of work lies in the design controller based on maximum sensitivity. The proposed design also uses higher order Pade's approximation for time delay. The closed loop performance is observed with nominal model, perturbed model and for noise in the measurement. The performance is computed using integral square error (ISE) and Integral absolute error (IAE). The controller effort is estimated using a measure called total variation (TV). Further, stability analysis is accomplished for variation in the model parameters and fragility analysis is carried out for uncertainties in the controller.

Keywords Non-minimum phase process · IMC filter · Performance · Robust stability · Fragility

1 Introduction

Control of non-minimum phase (NMP) processes are difficult due to the presence of right half plane (RHP) zeros. Some of the effects of RHP zeros on closed loop system are an undershoot in initial response, an overshoot in response and oscillations in the closed loop response. Such processes require more efforts for designing and tuning parameters of the controller. PID controller is still adopted in the industries owing to its simple design and lucid operation assuring accurate and stable performance of

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feedback loop. The significance of PID is its feasibility and easiness during implementation. There are wide range of PID tuning rules available for both stable [1, 2] and unstable systems [3–10].

IMC scheme is the mostly used to design the PID controller which explicitly uses model of the plant. An IMC-based fractional order PID (FOPID) controller design was proposed for NMP system with dead time by using stability inequality [11]. Also, an IMC-based fractional order controller was designed for predefined phase margin and gain crossover frequency and the performance was verified with DC-DC boost converter system [12]. A FOPID controller was designed for NMP processes where the controller settings are chosen using Nelder and Mead algorithm minimizing IAE [13].

In this article, an IMC-PID controller is proposed for a non-minimum second order plus time delay (SOPTD) system using higher order IMC filters to boost the controller's efficiency and robustness. The design incorporates different Pade's approximation for delay term in the NMP system. The derived controller contains a PID plus filter term. There will be only one parameter to be tuned in the designed controller which is the IMC filter time constant. The tuning is carried out for a specified maximum sensitivity (M_s) which assures robust performance. The NMP system performance with the designed controller is assessed based on minimum IAE and TV for actual parameters of the model and then by introducing perturbations in the parameters. It is necessary to know the system behavior for measurement noise and hence the closed loop response is also observed for white noise. Any controller is expected to be insensitive for parametric uncertainties in the model. This is verified through robustness analysis to figure out the robust stability and performance of the process [14, 15]. Practically, there is uncertainty in controller parameters too due to the tolerance of system components. The controller parametric uncertainties may affect the system performance leading to a situation of retuning the controller. To assess this situation, a fragility analysis is executed for the designed controller using M_s and IAE that deliver optimal and reliable control systems [16–20].

This paper is arranged as follows: Sect. 2 describes the IMC method and proposed controller design; Sect. 3 provides different analyses to check the system performance; Sects. 4 and 5 give the details about results and conclusion.

2 Controller Design

2.1 Internal Model Control Scheme

Figure 1 shows the IMC structure and close loop system. IMC scheme explicitly uses model $G_m(s)$ of process in the controller design. So, the IMC controller (Eq. 1) is

$$C_{IMC}(s) = \frac{1}{G_m^-(s)} f(s) \tag{1}$$

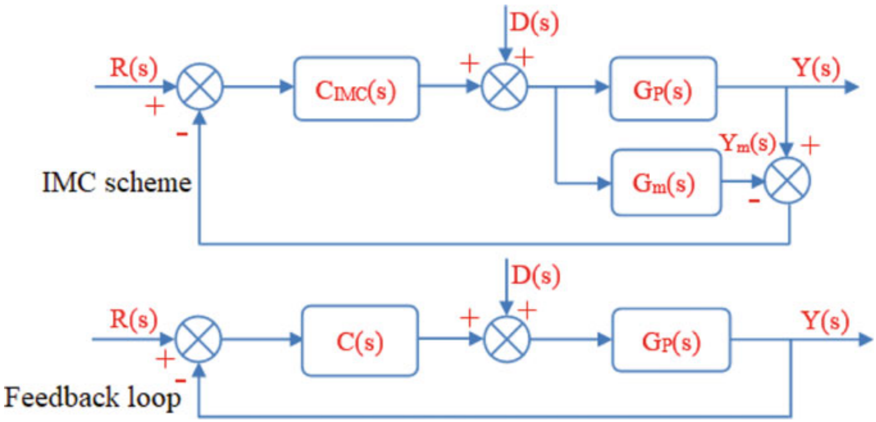

Fig. 1 Block diagram

$G_{\mathrm{m}}^{-}(s)$ is invertible part of the model and $f(s)$ is the IMC filter given by Eq. (2)

$$f(s) = \frac{1}{(\gamma s + 1)^n} \tag{2}$$

The 'n' and 'γ' in Eq. (2) are filter order and time constant, respectively. The equivalent feedback controller is given in Eq. (3).

$$C(s) = \frac{C_{\mathrm{IMC}}(\mathrm{s})}{1 - C_{\mathrm{IMC}}(\mathrm{s})G_{\mathrm{m}}(\mathrm{s})} \tag{3}$$

2.2 Proposed Controller Design

The proposed controller structure (Eq. 4) is

$$C(s) = (\text{integer filter term})K_{\mathrm{p}}\left[1 + \frac{1}{T_i s} + T_{\mathrm{d}}s\right] \tag{4}$$

The various parameters in Eq. (4) are proportional gain (K_{p}), integral time (T_{i}) and derivative time (T_{d}).

Consider a SOPTD-NMP model as given in Eq. (5).

$$G_m(s) = \frac{K(1 - T_a s)\mathrm{e}^{-Ls}}{(T_1 s + 1)(T_2 s + 1)} \tag{5}$$

where K—process gain, L—time delay, T_1 and T_2 are system time constants.

Table 1 Higher order IMC filter structures used for controller design

Filter order	IMC filters term
Second order	$\frac{1}{(\gamma s+1)^2}$
Third order	$\frac{1}{(\gamma s+1)^3}$
Fourth order	$\frac{1}{(\gamma s+1)^4}$

The present work employs higher order IMC filters listed in Table 1 for controller design as explained in Sect. 2.1.

The controller using model in Eq. (5) and IMC filters in Table 1 along with 1st order, 2nd order, 2/3rd order, 1/2 order Pade's approximations for time delay is given in Eq. (6).

$$C(s) = (\text{integer filter term})\frac{(T_1 + T_2)}{K}\left[1 + \frac{1}{(T_1 + T_2)s} + \frac{(T_1 T_2)}{(T_1 + T_2)}s\right] \quad (6)$$

The PID term of Eq. (6) will be consistent for all the proposed methods designed using different IMC filters and Pade's approximations for time delay and the variation will only be in the integer filter term (see Table 2 for corresponding filter terms).

3 Closed Loop Performance Analysis

The closed loop performance is analyzed with the measures ISE, IAE and TV for specific $M_{s.}$ These measures are presented in Eqs. (7)–(10).

$$\text{ISE} = \int_0^\infty \mathrm{e}^2(t)\mathrm{d}t \quad (7)$$

$$\text{IAE} = \int_0^\infty |\mathrm{e}(t)|\mathrm{d}t \quad (8)$$

$$\text{TV} = \sum_{i=0}^{\infty} |u_{i+1} - u_i| \quad (9)$$

$$M_s = \max_{0<\omega<\infty}\left|\frac{1}{1 + C(j\omega)G(j\omega)}\right| \quad (10)$$

Table 2 Integer filter terms for proposed controller

Method	Integer filter term
Proposed 1	$\frac{1+0.1s}{0.1\gamma^2 s^2+(\gamma^2+0.2\gamma-0.02)s+(2\gamma+0.4)}$
Proposed 2	$\frac{0.00332s^2+0.1s+1}{0.00332\gamma^2 s^3+(0.1\gamma^2+0.00664\gamma+0.000664)s^2+(\gamma^2+0.2\gamma-0.02)s+(2\gamma+0.4)}$
Proposed 3	$\frac{0.008s^3+0.36s^2+7.2s+60}{0.008\gamma^2 s^4+(0.36\gamma^2+0.016\gamma)s^3+(7.2\gamma^2+0.72\gamma+0.032)s^2+(60\gamma^2+14.4\gamma-0.72)s+(120\gamma+24)}$
Proposed 4	$\frac{0.04s^2+0.8s+6}{0.04\gamma^2 s^3+(0.8\gamma^2+0.08\gamma)s^2+(6\gamma^2+1.6\gamma-0.04)s+(12\gamma+2.4)}$
Proposed 5	$\frac{1+0.1s}{0.1\gamma^3 s^3+(\gamma^3+0.3\gamma^2)s^2+(3\gamma^2+0.3\gamma-0.02)s+(3\gamma+0.4)}$
Proposed 6	$\frac{0.00332s^2+0.1s+1}{0.00332\gamma^3 s^4+(0.1\gamma^3+0.00996\gamma^2)s^3+(\gamma^3+0.3\gamma^2+0.00996\gamma+0.000664)s^2+(3\gamma^2+0.3\gamma-0.02)s+(3\gamma+0.4)}$
Proposed 7	$\frac{0.008s^3+0.36s^2+7.2s+60}{0.008\gamma^3 s^5+(0.36\gamma^3+0.024\gamma^2)s^4+(7.2\gamma^3+1.08\gamma^2+0.024\gamma)s^3+(60\gamma^3+21.6\gamma^2+1.08\gamma+0.032)s^2+(180\gamma^2+21.6\gamma-0.72)s+(180\gamma+24)}$
Proposed 8	$\frac{0.04s^2+0.8s+6}{0.04\gamma^3 s^4+(0.8\gamma^3+0.12\gamma^2)s^3+(6\gamma^3+2.4\gamma^2+0.12\gamma)s^2+(18\gamma^2+2.4\gamma-0.04)s+(18\gamma+2.4)}$

(continued)

Table 2 (continued)

Method	Integer filter term
Proposed 9	$\frac{1+0.1s}{0.1\gamma^4 s^4+(\gamma^4+0.4\gamma^3)s^3+(4\gamma^3+0.6\gamma^2)s^2+(6\gamma^2+0.4\gamma-0.02)s+(4\gamma+0.4)}$
Proposed 10	$\frac{0.00332s^2+0.1s+1}{0.00332\gamma^4 s^5+\left(0.1\gamma^4+0.01328\gamma^3\right)s^4+\left(\gamma^4+0.4\gamma^3+0.01992\gamma^2\right)s^3+\left(4\gamma^3+0.6\gamma^2+0.01328\gamma+0.000664\right)s^2+\left(6\gamma^2+0.4\gamma-0.02\right)s+(4\gamma+0.4)}$
Proposed 11	$\frac{0.008s^3+0.36s^2+7.2s+60}{0.008\gamma^4 s^6+\left(0.36\gamma^4+0.032\gamma^3\right)s^5+\left(7.2\gamma^4+1.44\gamma^3+0.048\gamma^2\right)s^4+\left(60\gamma^4+28.8\gamma^3+2.16\gamma^2+0.032\gamma\right)s^3+\left(240\gamma^3+43.2\gamma^2+1.44\gamma+0.032\right)s^2\ \left(360\gamma^2+28.8\gamma-0.72\right)s+(240\gamma+24)}$
Proposed 12	$\frac{0.04s^2+0.8s+6}{0.04\gamma^4 s^5+(0.8\gamma^4+0.16\gamma^3)s^4+(6\gamma^4+3.2\gamma^3+0.24\gamma^2)s^3+\left(24\gamma^3+4.8\gamma^2+0.16\gamma\right)s^2+\left(36\gamma^2+3.2\gamma-0.04\right)s+(24\gamma+2.4)}$

3.1 Robustness Analysis

The stability of a closed loop system need to be analyzed for parametric uncertainty [21–23]. It is verified by robust stability condition given in Eq. (11):

$$l_m(j\omega)T(j\omega) < 1\forall\omega \in (-\infty, \infty) \tag{11}$$

$T(j\omega)$ is the complementary sensitivity function, $T(j\omega) = \frac{C(j\omega)G(j\omega)}{1+C(j\omega)G(j\omega)}$ and the process uncertainty bound is $l_m(j\omega) = \left|\frac{G(j\omega)-G_m(j\omega)}{G_m(j\omega)}\right|$. The controller must be tuned such that Eq. (12) is satisfied.

$$T(j\omega) < \frac{1}{\left|\left(\frac{\Delta K}{K}+1\right)e^{-\Delta L}-1\right|} \tag{12}$$

3.2 Fragility Analysis

It is necessary to assess the controller fragility as their exactness is not guaranteed during implementation. The robustness fragility index (RFI) is evaluated based on M_s using Eq. (13) for uncertainty in the controller parameters.

$$\text{RFI}_{\Delta\varepsilon} = \frac{M_{s\Delta\varepsilon}}{M_s} - 1 \tag{13}$$

where M_s—actual maximum sensitivity, $M_{s\Delta\varepsilon}$—new M_s for change in settings of the controller and ε is the parametric uncertainty.

The fragility of a controller is decided as follows: If $\text{RFI}_{\Delta 20} > 0.5$, the controller is fragile; if $0.1 < \text{RFI}_{\Delta}20 < 0.5$, it is non-fragile; and if $\text{RFI}_{\Delta}20 < 0.1$, it is resilient. Similarly, Eq. (14) is used to calculate the performance fragility index (PFI) based on IAE for variation in controller settings.

$$\text{PFI}_{\Delta\varepsilon} = \frac{\text{IAE}_{\Delta\varepsilon}}{\text{IAE}} - 1 \tag{14}$$

where IAE is the original value and $\text{IAE}_{\Delta e}$ is the IAE for change in settings of the controller.

In this work, a novel PFI is proposed based on ISE which is given in Eq. (15)

$$\text{PFI}_{\Delta\varepsilon} = \frac{\text{ISE}_{\Delta\varepsilon}}{\text{ISE}} - 1 \tag{15}$$

where ISE is the actual value and $\text{IAE}_{\Delta\epsilon}$ is the new ISE for change in controller settings.

The controller fragility is judged based on ISE and IAE as fragile, non-fragile and resilient for $\text{PFI}_{\Delta}20 > 0.5$, $0.1 < \text{PFI}_{\Delta}20 < 0.5$ and $\text{PFI}_{\Delta}20 < 0.1$ respectively.

4 Results and Discussion

4.1 Example

Consider the SOPTD-NMP process [11] given by Eq. (16):

$$G_p(s) = \frac{(1 - 0.2s)e^{-0.2s}}{(s+1)^2} \tag{16}$$

Initially, twelve controllers are designed using the model in Eq. (16) with the different IMC filters (Table 1) and Pade's approximation for time delay. Then, the nominal response is observed for step input and it was found that the proposed methods from 1 to 8 are resulting in oscillations in the response. Hence, Proposed1 to Proposed 8 methods are not included in the performance analysis. Only, Proposed9 to Proposed 12 controllers are used for simulation and the performance is analyzed for M_s=2. The PID settings for Proposed9, Proposed10, Proposed11 and Proposed12 methods are $K_p = 2$, $T_i = 2$ and $T_d = 0.5$. The tuning parameter γ values for Proposed9, Proposed10, Proposed11 and Proposed12 methods are chosen 0.0907, 0.08918, 0.08918 and 0.09053 respectively satisfyingM_s=2. The corresponding integer filter terms are shown in Table 3.

The step response for a change in set point and disturbance of magnitude 1 and 0.5 (given at 10 s.) is shown in Fig. 2 and the associated performance measures are listed in Table 4. It is noticed that Proposed10 and Proposed11 methods are superior in performance compared to Proposed9 and Proposed12 methods with low ISE, IAE and TV. Similarly, the closed loop response is observed for 15% perturbation in

Table 3 Integer filter terms

Method	Integer filter term
Proposed 9	$\frac{1+0.1s}{6.76\times10^{-6}s^4 + 3.66\times10^{-4}s^3 + 0.00792s^2 + 0.0656s + 0.7628}$
Proposed 10	$\frac{0.00332s^2 + 0.1s + 1}{2.099\times10^{-7}s^5 + 1.574\times10^{-5}s^4 + 5.0537\times10^{-4}s^3 + 0.009457s^2 + 0.06339s + 0.75672}$
Proposed 11	$\frac{0.008s^3 + 0.36s^2 + 7.2s + 60}{5.06\times10^{-7}s^6 + 4.547\times10^{-5}s^5 + 0.001858s^4 + 0.04425s^3 + 0.6742s^2 + 4.7115s + 45.403}$
Proposed 12	$\frac{0.04s^2 + 0.8s + 6}{2.687\times10^{-6}s^5 + 1.7245\times10^{-4}s^4 + 0.004744s^3 + 0.0716s^2 + 0.5447s + 4.5727}$

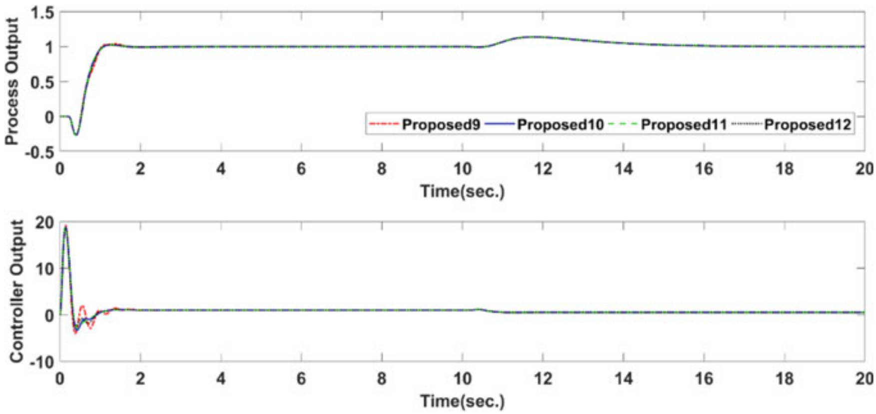

Fig. 2 Closed loop response with nominal model

Table 4 Performance measures for nominal process

Method	ISE	IAE	TV
Proposed 9	0.7777	1.18	61.3071
Proposed 10	0.7745	1.158	46.5358
Proposed 11	0.7745	1.157	45.6482
Proposed 12	0.7777	1.164	46.1296

all parameters of the model (Fig. 3) and for noise (mean = 0 and variance = 0.1) in output (Fig. 4). The corresponding ISE, IAE and TV values are listed in Table 5. Proposed10 and Proposed11 methods are showing better performance even with perturbations and noise than Proposed9 and Proposed12 methods.

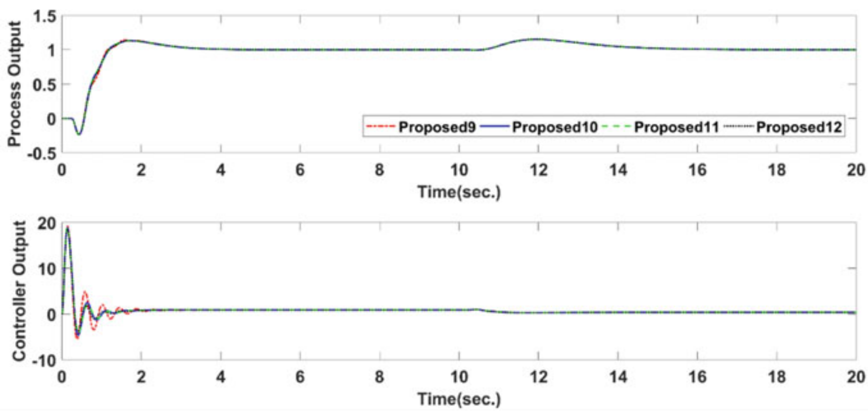

Fig. 3 Closed loop response for perturbations

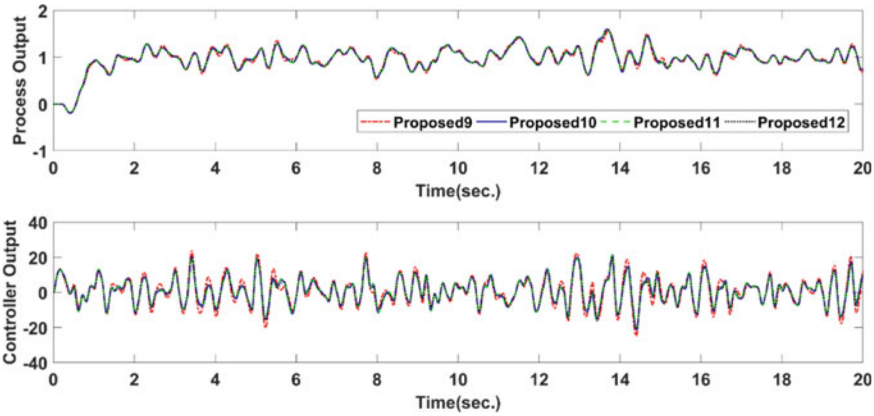

Fig. 4 Closed loop response for measurement noise

Table 5 ISE, IAE and TV values for perturbations and output noise

Method	Perturbed response			Noise response		
	ISE	IAE	TV	ISE	IAE	TV
Proposed 9	0.8438	1.406	78.5291	4.897	9.639	2952.6
Proposed 10	0.8381	1.395	57.6758	4.891	9.629	2568.0
Proposed 11	0.8379	1.395	54.6540	4.87	9.612	2559.1
Proposed 12	0.8415	1.401	53.4656	4.843	9.603	2563.5

The magnitude plot shown in Fig. 5 proves the system stability for +15% uncertainty in L and K. It is evident that the closed loop system is stable with the Proposed9, Proposed10, Proposed11 and Proposed12 methods fulfilling the robust stability condition (Eq. 12).

The RFI values for variation in controller settings are shown in Fig. 6. It is observed that the controllers are non-fragile up to +15% variation in controller settings and become fragile for +20% variation. Hence, one should see that the controller parameter variation should not increase beyond 15%.

The PFI values based on IAE and ISE are illustrated in Figs. 7 and 8. It is evident from the trends of PFI values based on IAE (Fig. 7) that the controllers are fragile for +20% variation in their settings. All the proposed methods are non-fragile for +20% variation in controller settings which is true from the trends of PFI based on ISE (Fig. 8). Hence, care should be taken while changing the controller settings as the proposed controllers are becoming fragile for more than +10% uncertainty.

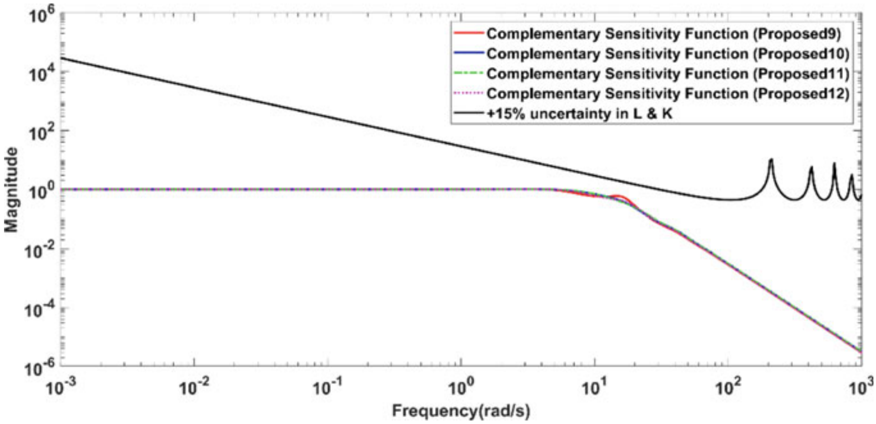

Fig. 5 Magnitude plot

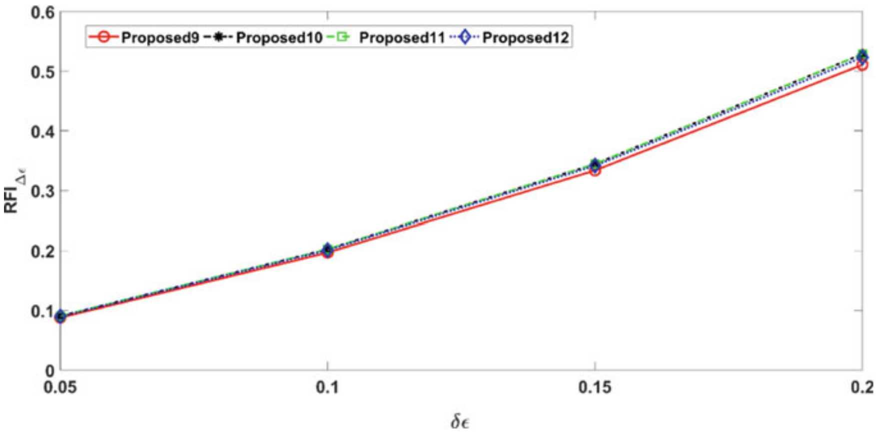

Fig. 6 Trends of RFI values based on M_s

5 Conclusion

In this article, an IMC-based PID controller is developed for non-minimum phase systems based on M_s. The optimum controller is selected based on minimum IAE for a predefined M_s. It is found that the controllers developed with fourth order IMC filter are giving stable response compared to second and third order IMC filters. The controller constructed with the optimum 4th order IMC filter and 2/3rd Pade's approximation for time delay shows improved response for nominal model parameters, model uncertainties and output noise. This is proved with low values of performance measures and control effort. It is found that all the proposed methods are stable for model uncertainties. All the proposed controllers are found to be non-fragile up

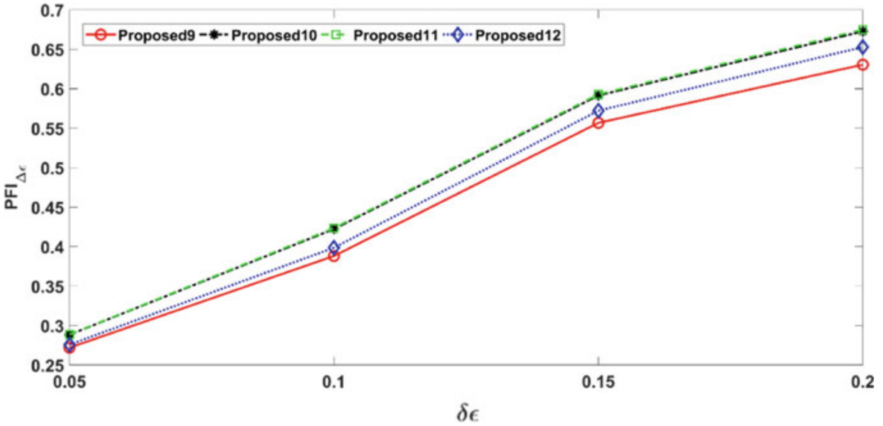

Fig. 7 Trends of PFI values based on IAE

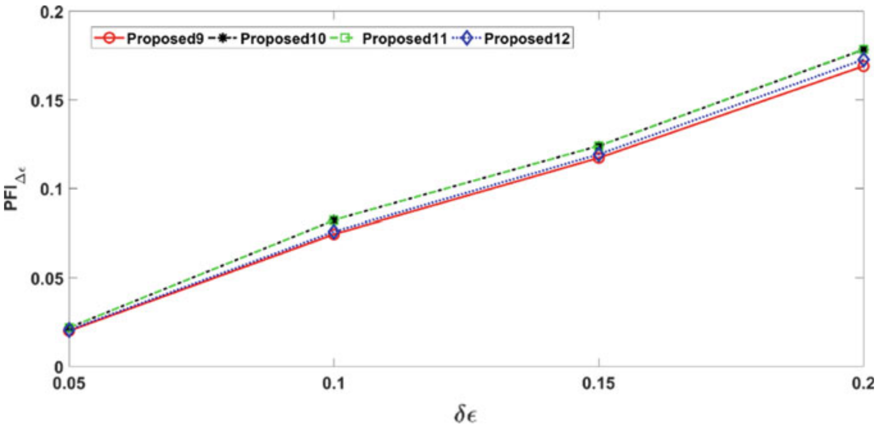

Fig. 8 Trends of PFI values based on ISE

to +20% variation in controller parameters based on ISE while they are fragile for +20% shift in controller settings based on M_s and ISE. Care should be taken while changing the controller settings as they affect system performance and robustness.

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