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Towards the Decentralized Energy Optimization of Active Unbalanced Distribution Systems Integrated with Natural Gas Systems

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***Abstract*—This article presents the ADMM-based decentralized multi-period energy optimization of an active unbalanced power distribution system integrated with the natural gas distribution network. The second-order cone programming (SOCP) based relaxations are utilized to convexify the nonconvexities involved in the multi-energy system. The relaxation error in each subsystem problem is iteratively reduced to predefined accepted levels. The power distribution subsystem problem is modeled as a bi-level problem. Level 1 solves the linearized version, and level 2 solves the nonlinear version of the subsystem problem. The sequential bound tightening algorithm and solution recovery procedures are proposed to recover meaningful solutions to the original nonconvex subsystems problem from their relaxed counterparts. The proposed solution procedure minimizes the multi-period operation cost of the multi-energy system. The simulations are conducted on an active unbalanced IEEE-13 bus distribution network and a 6-node natural gas distribution network. The results obtained corroborate the proposed solution strategy.**

***Index Terms*—ADMM, multi-energy system, natural gas, SOCP, three-phase optimal power flow, unbalanced distribution system**

NOMENCLATURE

Acronyms

UDS	Unbalanced Distribution System
NGD	Natural Gas Distribution
IGDS	Integrated Natural Gas and Distribution System
NG	Natural Gas
OPF	Optimal Power Flow
BFM	Branch Flow Model
SOCP	Second Order Cone Programming
MINLP	Mixed-Integer Non Linear Programming
ADMM	Alternating Direction Method of Multipliers

Parameters and Variables

$p_{k,t}^{a,L}/q_{k,t}^{a,L}$	kW/kVAr power load for phase a
z_{ij}	Three-phase Z matrix for line (j,k)
z_{jk}^{ab}	$z_{jk}^{ab} = r_{jk}^{ab} + jx_{jk}^{ab}$ is a Z matrix element
$G_{i,t}^{L}$	NG load demand at node i, time t
$\gamma_1, \gamma_2, \gamma_3$	NG fired DGs cost curve coefficients
$\underline{v}_k,\bar{v}_k$	Voltage boundary conditions at bus k
pr_i^{min}, pr_i^{max}	Min/max nodal pressure limits in NGD
C_{ij}	NGD pipeline (i,j) parameter
v_k^a	voltage square variable for phase a
l_{jk}^a	Phase a current square variable in line (j,k)
$p_{k,t}^{g,a}/q_{k,t}^{g,a}$	Substation kW/kVAr power generation for phase a at bus k
$q_{k,t}^{a,c}$	Capacitor power injection for phase a located at bus k
$p_{k,t}^{a,dg}/q_{k,t}^{a,dg}$	NGDG kW/kVAr generation for phase a situated at node k
$p_{k,t}^{pv}/q_{k,t}^{pv}$	Solar kW/kVAr generation at node k
$S_{jk,t}^{aa}/p_{jk,t}^{aa}/q_{jk,t}^{aa}$	kVA/kW/kVAr power flows in phase-a of branch (j,k)
$G_{ij,t}$	NG flow in gas pipeline (i,j)
$G_{i,t}^{S}$	NG supplied at node i
$G_{i,t}^{dg}$	NG consumed by NGDG located at node i
pr_i	NG nodal pressure at node i

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I. INTRODUCTION

Compared to the individual energy supply system, the integrated multi-energy system has become a more efficient and flexible energy optimization system, which can cater to the multi-type of load demands such as gas, heat, and electricity [1]. Asl et al., in [2], focus on the operation of an integrated energy system comprising an unbalanced distribution system, natural gas, and heating system. However, instead of a recent classical approach like SOCP, SDP, etc., the metaheuristic optimization solution technique is proposed as the solution strategy. The classical relaxation techniques like SOCP, SDP, McCormick envelopes, chordal relaxations, etc., solving NP-hard optimization problems are gaining attention. These techniques are used to solve the optimal power flow (OPF) in electricity and gas flow problems in natural gas networks. However, their inexactness in solving unbalanced distribution systems brings additional computational challenges to the system operator. The consensus-based alternating directional method of multipliers (ADMM) approach is proposed in [3] to minimize the operational cost of both electricity and gas networks. However, the DC power flow-based electricity modeling and transmission level system operation are considered. The day-ahead system operation of integrated electricity and natural gas at the distribution level considering the uncertainty of gas loads is devised in [4]. The distribution system is assumed to be a balanced system; however, in reality, the distribution system is unbalanced due to unsymmetrical distribution lines, unbalanced load, mutual couplings, etc. The co-optimization of integrated gas and electricity networks is extensively studied at the transmission level compared to the distribution levels. The complex energy relationship among coupling facilities and the inherent differences in multi-energy network structures bring challenges to the energy optimization of integrated natural gas and distribution systems (IGDS). Moreover, with the renewable energy penetration and the unbalanced nature of the electricity distribution network, the multi-period operation of IGDS faces higher computational challenges. Several studies have focused on proposing techniques for relaxation of constraints related to the coupling facilities, which include lagrangian relaxation [5], alternating directional method of multipliers (ADMM) [6], a convex relaxation model [7], and benders decomposition [8].

The unbalanced three-phase distribution system (UDS) with voltage control devices such as capacitor banks, voltage regulators, and coupling facilities results in a mixed-integer nonlinear program (MINLP). Even with the recent advancement in solving three-phase optimal power flow (OPF), the OPF problem with associated nonlinearities and mutual phase coupling with only continuous variables is quite challenging. This article addresses this problem by solving the electrical subsystem problem using a bi-level approach. The inherent nonconvexities in the IGDS are relaxed using the SOCP relaxation. The solutions to the original nonlinear subproblems are then iteratively recovered by shrinking the relaxation error to the accepted levels.

According to the above research discussion, the main contributions of this work are listed as follows:

1) The UDS problem formulation is modeled as a bi-level programming problem. Where level 1 solves the MILP subproblem of the UDS subsystem to determine the control settings of the voltage control devices. The nonlinear programming problem in level 2 is devised as a convex NLP problem, giving a lower bound to the objective function.
2) The nonlinearities in both subsystems are addressed using SOCP relaxations. The feasible and optimal solution for each subsystem is recovered from their relaxed counterparts by reducing the relaxation errors below the predefined tolerance levels.
3) The overall IGDS problem is solved using an ADMM-based decentralized solution approach sharing minimal information while respecting the UDS and NGD privacy concerns.

The rest of the article is organized as follows. The problem formulation of UDS and DHS and their objective functions are described in Section II. The ADMM-based decentralized solution technique aiming to minimize IGDS operation cost while reducing the relaxation error is introduced in Section III. The simulation studies and result discussions are shown in Section IV, and the conclusions are drawn in Section V.

II. PROBLEM FORMULATION

A. Objective function of IGDS

The objective of IGDS is to minimize the operating costs of both the Natural gas distribution (NGD) and the unbalanced three-phase distribution system (UDS). The objective function is expressed as follows

$$OF = \sum_{t \in T} [Cost_t^{UDS} + Cost_t^{NGD}] \tag{1}$$

Where, the overall cost of operation of an electrical system includes the cost of power purchased from the main grid and the cost of NG fired generating units DGs. The operation cost of NGD is expressed as the cost of total gas supplied.

B. UDS Modeling

The Branch Flow Model (BFM) based three-phase real and reactive power balance equations are expressed as [9]

$$\begin{aligned} p_{jk,t}^{aa} - \sum_{b \in \phi_k} l_{jk,t}^{ab} (r_{jk}^{ab} cos(\theta_{jk,t}^{ab} + x_{jk}^{ab} sin(\theta_{jk,t}^{ab})) = \sum_{l} p_{kl,t}^{aa} \\ + p_{k,t}^{a,L} - (p_{k,t}^{a,g} + p_{k,t}^{a,dg} + p_{k,t}^{a,pv}) \end{aligned} \tag{2}$$

$$\begin{aligned} q_{jk,t}^{aa} - \sum_{b \in \phi_k} l_{jk,t}^{ab} (x_{jk}^{ab} cos(\theta_{jk,t}^{ab} - r_{jk}^{ab} sin(\theta_{jk,t}^{ab})) = \sum_{l} q_{kl,t}^{aa} \\ + q_{k,t}^{a,L} - (q_{k,t}^{a,g} + q_{k,t}^{a,c} + q_{k,t}^{a,dg} + q_{k,t}^{a,pv}) \end{aligned} \tag{3}$$

where, p_{jk}^{aa} and q_{jk}^{aa} are real & reactive power flows in phase-a of line $(j,k) \in \mathcal{L}$ at time t, respectively. $r_{jk}^{ab} + jx_{jk}^{ab}$ is the mutual impedance of line (j,k) between phases a & b. $\phi_k = \{a,b,c\}$ The voltage drop equation is expressed as

$$\begin{aligned} v_{k,t}^{a} - v_{j,t}^{a} = -\sum_{b \in \phi_k} 2\,\mathrm{Re}\{[S_{jk,t}^{ab} (z_{jk}^{ab})^{*}]\} + \sum_{b \in \phi_k} |z_{jk}^{ab}|^2 l_{jk,t}^{bb} \\ + \sum_{b1,b2 \in \phi_k, b1 \neq b2} 2\,\mathrm{Re}\Big\{\Big[z_{jk}^{ab1} l_{jk,t}^{b1b2} \left(\angle(\theta_{jk,t}^{b1b2})\right) \left(z_{jk}^{ab2}\right)^{*}\Big]\Big\} \end{aligned} \tag{4}$$

The relationship between line power flow, current and voltage is given as

$$(p_{jk,t}^{aa})^2 + (q_{jk,t}^{aa})^2 = v_{j,t}^{a} l_{jk,t}^{aa}, \qquad (l_{jk,t}^{ab})^2 = l_{jk,t}^{aa} l_{jk,t}^{bb} \tag{5}$$

The forecasted real power output at time t of a smart inverter connected solar panels placed at node k is expressed as [10]

$$p_{k,t}^{pv} = \eta_{pv} A_k^{pv} Ir_t (1 - 0.005(T_t^{amb} - 25)) \tag{6}$$

The reactive power flow to/from the smart PV inverter is constrained by (7)

$$q_{k,t}^{pv} \leq \sqrt{(\bar{S}_k^{pv})^2 - (p_{k,t}^{pv})^2} \tag{7}$$

The SOCP relaxation of constraints (5) are expressed as

$$(p_{jk,t}^{aa})^2 + (q_{jk,t}^{aa})^2 \leq v_{j,t}^{a} l_{jk,t}^{aa} \quad \& \quad (l_{jk,t}^{ab})^2 \leq l_{jk,t}^{aa} l_{jk,t}^{bb} \tag{8}$$

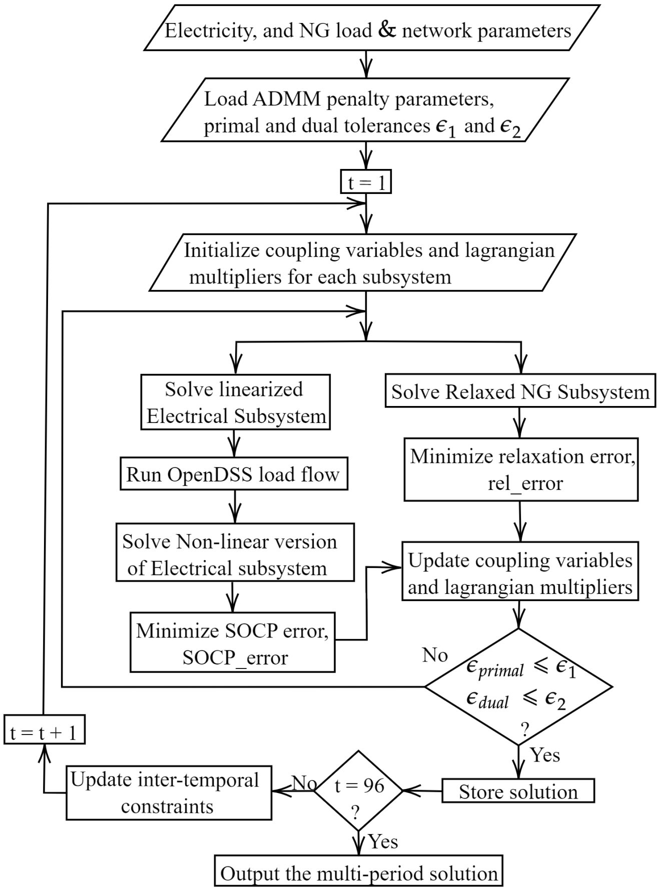

Fig. 1. Flowchart of Solution Approach

C. NGD Modeling

For NGD modeling, it is assumed that the pipeline gas flow directions are known beforehand and, therefore, fixed [11]. Moreover, the compressors need not be installed for distribution networks compared to the transmission levels where the pressure loss is sizeable [12]. The Weymouth equation, gas flow continuity constraint, and boundary conditions on the gas pipeline are expressed in (9)-(11).

$$G_{ij,t}^2 = C_{ij}(pr_{i,t}^2 - pr_{j,t}^2) \tag{9}$$

$$G_{ij}^{min} \leq G_{ij,t}^2 \leq G_{ij}^{max}, \qquad pr_i^{min} \leq pr_i^2 \leq pr_i^{max} \tag{10}$$

$$\sum_{ij \in \mathcal{P}} G_{ij,t} + G_{i,t}^S - G_{i,t}^L - G_{i,t}^{dg} = 0 \tag{11}$$

Where, $G_{ij,t}$ is the gas flow in pipeline $(i,j) \in \mathcal{P}$. C_{mn} is the pipeline parameter.

The constraints related to the interconnected coupling facility i.e. NG fired generating units between UDS and NGD is formulated as

$$G_{i,t}^{dg} \geq (1/\mu)(\gamma_1(p_{k,t}^{dg})^2 + \gamma_2 p_{k,t}^{dg} + \gamma_3), \qquad p_{k,t}^{dg} = \sum_{a \in \phi_k} p_{k,t}^{a,dg} \tag{12}$$

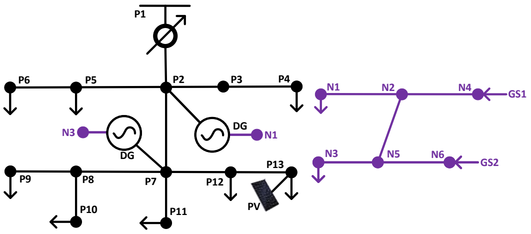

Fig. 2. Illustration of IGDS System

where $\gamma_1(MBtu/MWh^2)$, $\gamma_2(MBtu/MWh)$ and $\gamma_3(MBtu)$ are heat curve coefficients. μ is MBtu to kcf conversion factor.

III. Solution Strategy

This section explains an ADMM-based decentralized energy optimization solution strategy of IGDS, and the overall solution approach is shown in figure 1. The UDS subsystem problem is solved using a bi-level approach where level-1 solves the linearized version and level-2 solves the nonlinear convex relaxation of the UDS subsystem. The line current angles computed using the OpenDSS load flow based on the information received from level-1 are, therefore, treated as constant values for the level-2 problem. The solution obtained from each subsystem's convex relaxed version might not be meaningful to their original nonlinear counterparts. Hence, the solution is iteratively recovered by contracting the relaxation error for each subsystem.

IV. Results and Discussion

The simulations are performed on a computer system with Intel(R) Core(TM) i5-7400 CPU @3GHz & 16 GB RAM installed. The proposed solution approach is coded in GAMS Distribution 37.1.0 integrated with MATLAB 2019b. The IGDS system multi-period optimization problem is solved using a BARON solver with GAMS distribution.

The topology of the IGDS system with coupling facilities and renewable energy sources is shown in Fig. 2. The multiple energy system IGDS comprises a 6-node natural gas distribution system [13] and an unbalanced IEEE-13 bus distribution system.
Fig. 4 shows the evolution of the error in dual variables as the number of iterations progresses. This shows the consensus among coupling facilities of UDS and IGDS subsystems.
The electricity and natural gas load demand curves are shown in Fig. 3(a). The capacity of the solar panels is considered 50% of the rated load at the respective phase of the buses. And the size of the PV inverter is set by considering the power factor of 0.9.
Fig. 3(b) shows the three-phase voltages of the substation node and phase-c of a leaf node. It can be observed that the

voltages for each phase are within the boundary limits of ±5% from 1.0 pu. Fig. 3(c) shows the natural gas supplied by each gas supplier. The gas supplier GS2 dominates the GS1 in respect of the amount of gas supplied due to its low supplying cost. However, GS2 couldn't supply up to its total capacity due to the pipeline $(5, 2)$ gas flow boundary limits. Fig. 3(d) depicts the penetration of the NG-fired DGs power in the UDS system.

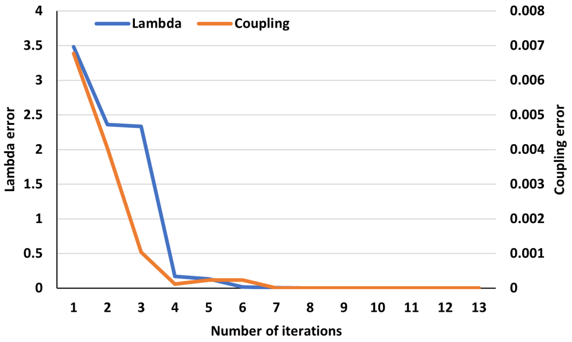

Fig. 4. Lambda and coupling error

The control settings for the power distribution network are shown in Table I. The voltage and substation power control settings of UDS corresponding to the peak and off-peak hours are validated using the OpenDSS load flow solution.

TABLE I
MULTI-PERIOD OPTIMIZATION RESULTS FOR UDS

IEEE-13 bus	Peak Load			Minimum Load		
OPF settings from GAMS						
Phase	a	b	c	a	b	c
Regulator Tap	0	0	2	26	26	31
Capacitor1 Status	OFF	OFF	OFF	OFF	OFF	OFF
Capacitor2 Status	-	-	OFF	-	-	OFF
Subst.Power (MW)	1.008	0.67	1.051	0.463	0.2877	0.483
Min Volt.(pu)	0.95	0.956	0.95	0.977	0.96	0.976
Max Volt. (pu)	1.049	1.05	1.05	1.043	1.039	1.05
DG1 (kW)	108.2	108.2	108.2	137.7	137.7	137.7
DG2 (kW)	100.4	100.4	100.4	108.7	108.7	108.7
Voltages and Substation Load Flow Validation Using OpenDSS						
Subst.Power (MW)	1.01	0.66	1.03	0.464	0.2883	0.484
Min Volt. (pu)	0.95	0.958	0.95	0.976	0.958	0.97
Max Volt. (pu)	1.041	1.043	1.048	1.042	1.04	1.05

The economic benefits realized for the UDS subsystem are shown in Fig.5. The cost curves in Fig.5 correspond to the power delivered by the NG-fired distributed generators and the electricity cost for the same amount of power purchased from the substation when the UDS is operated individually, i.e., without integration. The gap between these two costs shows the economic benefits for the UDS subsystem. Therefore, integrating different energy systems facilitates technical and

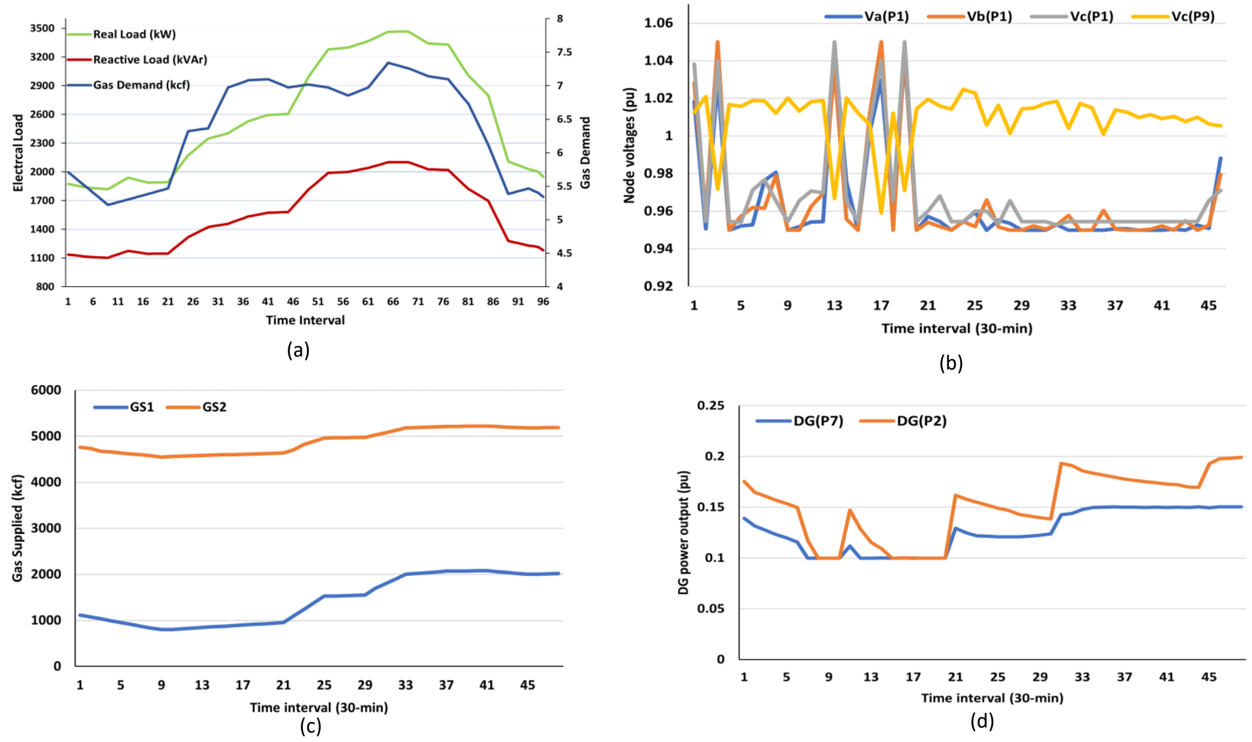

Fig. 3. (a) Load demand. (b) Node voltages. (c) Natural Gas supplied. (d) NG fired DGs power output.

economic advantages and brings more flexibility to the IGDS. Further, with the help of coupling facilities, e.g., distributed generation, the integrated energy system alleviates the power congestion problems in distribution lines. Also, IGDS facilitates the accommodation of new loads without further investment planning cost of distribution systems.

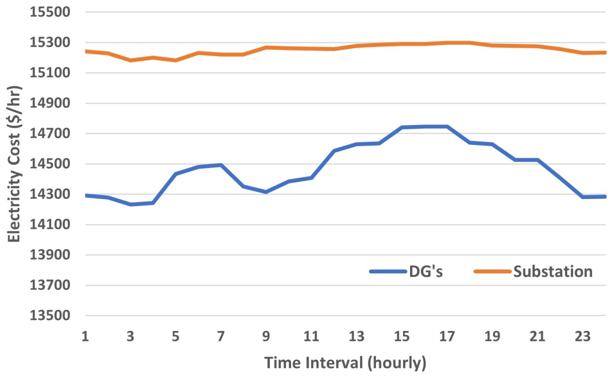

Fig. 5. Economic benefits of integrated system

V. CONCLUSION

This paper proposes an ADMM-based decentralized energy operation of an unbalanced distribution network inteconnected with the NG distribution system. The solution of the original nonconvex subproblem for each subsystem is iteratively retrieved from their relaxed counterparts. In each time interval for each subsystem, the error in dual variables is minimized iteratively until the predefined acceptance levels. The level-1 subproblem of the UDS subsystem computes the settings of the voltage control devices and coupling facilities to compute the line phase current angles. The results analysis demonstrates the effectiveness of the proposed solution technique, where phase voltages are well within limits in UDS subsystem and the gas pipeline flow limits in NGD subsystem. Apart from the technical and economic benefits, the integrated IGDS system brings more flexibility to the subsystems. The technical aspects, like the phase voltage unbalance rate in UDS after extensive integration with the NGD network, will be the focus of the future study.

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