Fragility evaluation of Integer order controller under process and controller parametric uncertainties

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***Abstract* — PID controller is dominantly used in the industries even today due to the availability of lucid tuning rules and simple structure. There were many analytical tuning rules proposed for stable time delay systems. Very few of them guarantee the robustness and performance of the closed loop system for process parametric uncertainties. It is also proved that PID controller should assure robust performance for perturbations in the controller parameters. It is interesting to note that whether the controller would assure robustness for perturbations in both the process parameters and controller parameters. Hence, the present work investigates the controller fragility for perturbations in both process and controller parameters.**

Keywords — Fragility, performance, maximum sensitivity, perturbation, robust stability

I. INTRODUCTION

PID controller is widely used in industries due to the simple and universally accepted tuning rules [1] and their ability to handle practical processes. There are mainly two types of PID controller tuning rules available in the literature such as rule based [2] and model based tuning rules [3], [4], [5], [6]. They were developed by using several techniques: step response test method, direct synthesis and internal model control (IMC) technique [7]. These tuning rules may not guarantee the performance due to several reasons including poor modeling and model uncertainties. However, IMC based technique is widely accepted for developing PID tuning rules as it explicitly considers the process model and offer flexibility in tuning. Many tuning rules were developed by minimizing error criteria and only some tuning rules considered robustness [8], [9]. Incorporating robustness (maximum sensitivity, M_s) in the controller design promises robust performance for uncertainties in the model.

The stability of a closed loop system can be assessed for using robust stability analysis [10] for perturbations in the process parameters. When a closed loop system is proved to be robustly stable for perturbations in the process parameters, the controller used in the feedback loop must also be robust for perturbations in its own parameters withstanding the perturbations in process parameters. With this motivation fragility analysis is carried out in this work for perturbations in not just controller parameters but also for perturbations in process parameters. The present work is limited to the fragility evaluation of integer order controller namely PID controller designed for stable first order plus time delay (FOPTD) and second order plus time delay (SOPTD) systems. The work is carried out by considering three proven PID controller tuning rules [8], [9], [4] based on robustness (M_s) for first and second order systems.

The article is organized as follows: The methodology used for evaluating the controller fragility is described in section II. The results are discussed in section III followed by conclusions in section IV.

II. METHODOLOGY

The transfer functions of FOPTD and SOPTD process models are:

$$G_1(s) = \frac{Ke^{-Ls}}{Ts+1} \tag{1}$$

$$G_2(s) = \frac{Ke^{-Ls}}{(T_1s+1)(T_2s+1)} \tag{2}$$

Where K and L are gain and time delay; T, T_1 and T_2 are time constants.

The parallel and series form of PID controller structures used for FOPTD and SOPTD systems are:

$$\text{Parallel form, } C(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right) \tag{3}$$

$$\text{Series form, } C(s) = K_c\left(1+\frac{1}{T_i s}\right)(1+T_d s) \tag{4}$$

Where K_c is the proportional gain, T_i is the integral time and T_d is the derivative time

The parallel form of PID was used in [8]; methods in [9] and [4] have used series form of PID controller.

The expression used to evaluate robustness fragility index (RFI) for perturbations in controller parameters [11], [12] is

$$\text{Robustness fragility index, } RFI_{\Delta\varepsilon} = \frac{M_{s\Delta\varepsilon}}{M_s}-1 \tag{5}$$

Where $\Delta\varepsilon = f\{\Delta K_c, \Delta T_i, \Delta T_d\}$; M_s is the nominal maximum sensitivity defined as

$$\max_{0<\omega<\infty}\left|\frac{1}{1+C(j\omega)G(j\omega)}\right| \tag{6}$$

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$M_{s\Delta\varepsilon}$ is the maximum sensitivity value for variation in controller parameters

The present work calculates the RFI values by considering $\Delta\varepsilon$ as a function of both process parameters and controller parameters i.e., $\Delta\varepsilon = f\ \{\Delta K, \Delta L, \Delta T, \Delta K_c, \Delta T_i, \Delta T_d\}$. Finally, controller fragility is judged by using the RFI values obtained for +20% variations in both process parameters and controller parameters. The criteria for judging the controller fragility is: fragile if $RFI_{\Delta 20} > 0.5$; nonfragile if $RFI_{\Delta 20} \leq 0.5$ and resilient if $RFI_{\Delta 20} \leq 0.1$. A resilient controller means that the loss of robustness, M_s is less than 10% of its nominal value for +20% variations in process and controller parameters. Nonfragile controller means there is 10% to 50% loss in M_s for +20% variations. In this case, it is possible to retune the controller to achieve the desired performance. If the loss in M_s value is more than 50% stable performance can never be achieved and there is no way to retune the controller.

The procedure followed to estimate the controller fragility for perturbation in both process and controller parameter is presented as follows:

Step 1: Identify the maximum possible perturbation in the process model for which the closed loop system becomes robustly stable. The robust stability condition [10] for perturbation in K and L is

$$\|T(j\omega)\|_\infty < \frac{1}{\left|\left(\frac{\Delta K}{K}+1\right)e^{-\Delta L}-1\right|} \tag{7}$$

$T(j\omega)=\frac{C(j\omega)G(j\omega)}{1+C(j\omega)G(j\omega)}$ is the complementary sensitivity function.

Step 2: Calculate RFI values for +5% variations in all process and controller parameters

Step 3: Record RFI values upto perturbation of +25% in all process parameters and controller parameters by increasing perturbation in steps of +5%.

Step 4: Next, calculate RFI values for +5% perturbations in all process parameters and single controller parameter.

Step 5: Repeat step 3 but with perturbation in only K_c.

Step 6: Repeat step 5 with perturbation in only T_i and then T_d.

Step 7: Repeat steps 4 to 6 for perturbations in any two controller parameters along with perturbations in process parameters.

Step 8: Identify which controller parameter/controller parameters are causing controller fragility when there are perturbation in both controller parameters and process parameters.

III. RESULTS AND DISCUSSION

The PID controller tuning rules proposed by methods in [8], [9], [4] have been considered for identifying the fragility nature of the controller for perturbation in model parameters and controller parameters. Initially, the robust stability is assessed through magnitude plot for perturbations in the process parameters. Fig. 1 shows the magnitude plot for +25% perturbations in K and L for both FOPTD and SOPTD systems. It is observed that the closed loop system is robustly stable for the three tuning rules studied in this work. The controller fragility for +25% perturbations in process parameters for the two examples is discussed with the following examples.

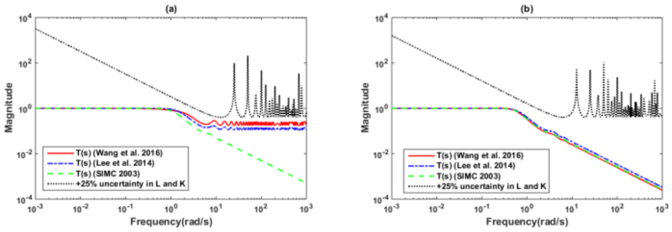

Fig.1. Magnitude plot for (a) FOPTD system (b) SOPTD system

A. Example 1

The FOPTD process model [8] used here is

$$G(s) = \frac{e^{-s}}{0.2s+1} \tag{8}$$

The controller settings for methods in [8], [9] and [4] are: $K_c = 0.347$, $T_i = 0.567$, $T_d = 0.12$; $K_c = 0.125$, $T_i = 0.2$, $T_d = 0.2$; and $K_c = 0.1$, $T_i = 0.2$. These settings were calculated for M_s value of 1.6. The fragility plot for variation in all process parameters and controller parameters is shown in Fig. 2. The $RFI_{\Delta 20}$ values are shown in Fig. 3. It is observed from these two figures that all the three methods are nonfragile.

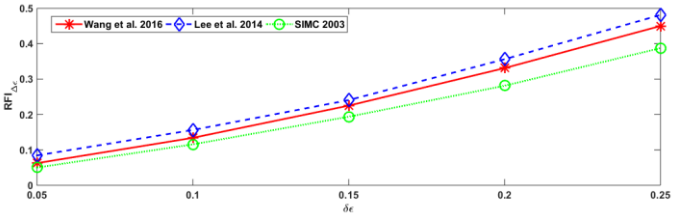

Fig. 2. Fragility for variation in all process parameters and controller parameters

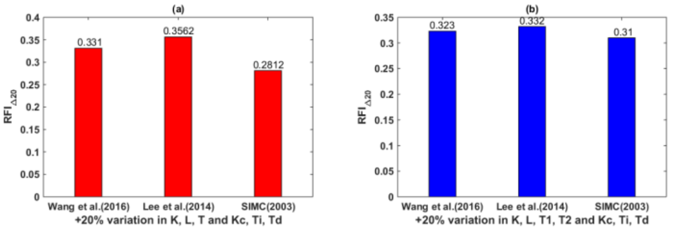

Fig. 3. $RFI_{\Delta 20}$ values (a) for FOPTD system (b) SOPTD system

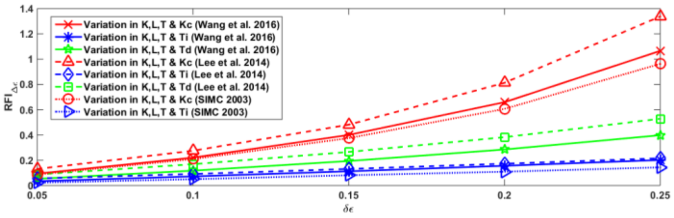

Fig. 4. Fragility for variation in all process parameters and single controller parameter

The fragility plot for variation in all process parameters and single controller parameter is shown in Fig. 4 and the corresponding $RFI_{\Delta 20}$ values are shown in Fig. 5. The controller becomes nonfragile for variation in T_i and T_d whereas it becomes fragile for variation in K_c. The fragility plot for variation in all process parameters and any two controller parameters is shown in Fig. 6. The associated $RFI_{\Delta 20}$ values are shown in Fig. 5. The controller becomes fragile for variation in K, L, T, K_c and T_d and nonfragile for variation in combination of K_c, T_i and T_i, T_d. It is quite evident that the PID controller is becoming fragile for perturbations in all process parameters, perturbation in K_c and the combination of K_c and T_d. The $RFI_{\Delta 20}$ value is low for method in [4] followed by methods in [8] and [9]. The controller fragility for the three methods is summarized in Table I and Table II.

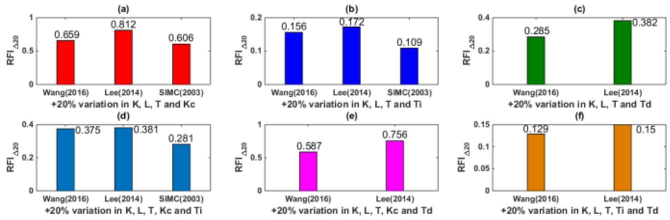

Fig. 5. (a) to (c): $RFI_{\Delta 20}$values for variation in all process parameters and single controller parameter; (d) to (f): $RFI_{\Delta 20}$ values for variation in all process parameters and two controller parameters

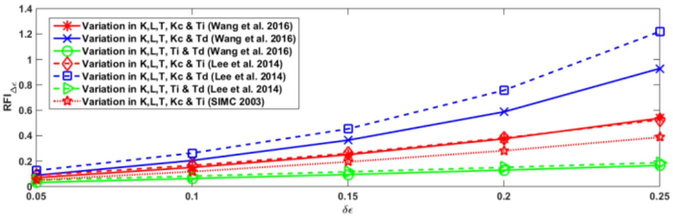

Fig. 6. Fragility for variation in all process parameters and two controller parameters

TABLE I NATURE OF FRAGILITY FOR VARIATION IN ALL PROCESS PARAMETERS AND SINGLE CONTROLLER PARAMETER

Method	***K_c variation***	***T_i variation***	***T_d variation***
Method in [8]	Fragile	Nonfragile	Nonfragile
Method in [9]	Fragile	Nonfragile	Nonfragile
Method in [4]	Fragile	Nonfragile	-

TABLE II NATURE OF FRAGILITY FOR VARIATION IN ALL PROCESS PARAMETERS AND TWO CONTROLLER PARAMETERS

Method	***K_c & T_i variation***	***K_c & T_d variation***	***T_i & T_d variation***
Method in [8]	Nonfragile	Fragile	Nonfragile
Method in [9]	Nonfragile	Fragile	Nonfragile
Method in [4]	Nonfragile	-	-

A. Example 2

The SOPTD model [8] used for the study is

$$G(s) = \frac{e^{-2s}}{(s+1)(0.7s+1)} \tag{9}$$

The controller settings with M_s=1.64 for methods in [8], [9] and [4] are: K_c = 0.435, T_i = 1.653, T_d = 0.4; K_c = 0.272, T_i = 1, T_d=0.86; and K_c=0.263, T_i=1, T_d=0.7. The fragility plots for variation in all process parameters and all controller parameters are shown in Fig. 7. Fig. 8 and Fig. 9 presents the fragility plots for perturbation in single controller parameter and any two controller parameters along with perturbation in all process parameters. The $RFI_{\Delta 20}$ values for the above three cases are shown as bar chart in Fig. 3 and Fig. 10. The three PID tuning methods used in this work have become nonfragile for all parameter variation both in model and controller. Similarly, the controller has become fragile for perturbation in all model parameters and K_c; perturbation in all process parameters, K_c and T_d. It becomes nonfragile for perturbation in other combinations of the controller parameters which is evident from Figs. 8 -10. The fragile and nonfragile nature of the PID controller for the three methods under study is listed in Table III and Table IV.

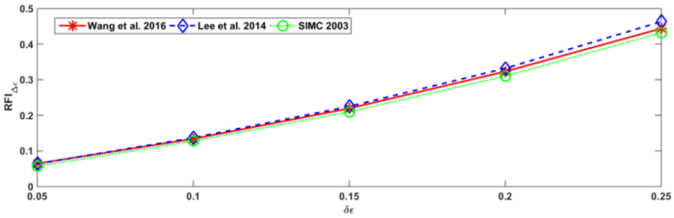

Fig. 7. Fragility for variation in all process parameters and controller parameters

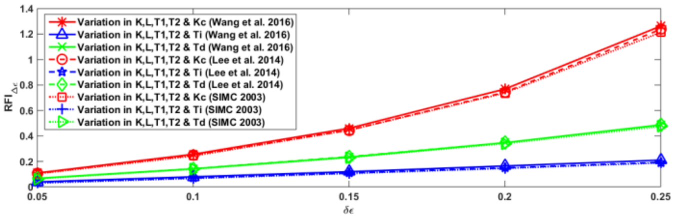

Fig. 8. Fragility for variation in all process parameters and single controller parameter

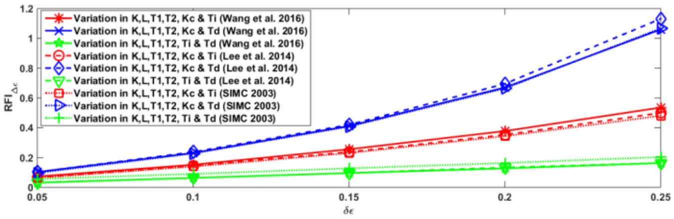

Fig. 9. Fragility for variation in all process parameters and two controller parameters

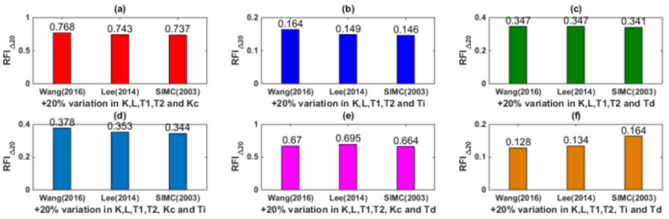

Fig. 10. (a) to (c): $RFI_{\Delta 20}$ values for variation in all process parameters and single controller parameter; (d) to (f): $RFI_{\Delta 20}$ values for variation in all process parameters and two controller parameters

TABLE III NATURE OF FRAGILITY FOR VARIATION IN ALL PROCESS PARAMETERS AND SINGLE CONTROLLER PARAMETER

Method	***K_c* variation**	***T_i* variation**	***T_d* variation**
Method in [8]	Fragile	Nonfragile	Nonfragile
Method in [9]	Fragile	Nonfragile	Nonfragile
Method in [4]	Fragile	Nonfragile	Nonfragile

TABLE IV NATURE OF FRAGILITY FOR VARIATION IN ALL PROCESS PARAMETERS AND TWO CONTROLLER PARAMETERS

Method	***K_c & T_i* variation**	***K_c & T_d* variation**	***T_i & T_d* variation**
Method in [8]	Nonfragile	Fragile	Nonfragile
Method in [9]	Nonfragile	Fragile	Nonfragile
Method in [4]	Nonfragile	Fragile	Nonfragile

IV. CONCLUSION

The fragility of PID controller is evaluated for perturbations in not just controller parameters but also for perturbation in process parameters of stable processes. The allowable process parameter perturbation is identified through robust stability analysis. The fragility is calculated for three scenarios:

1) Perturbation in all process parameters and all controller parameters

2) Perturbation in all process parameters and any one controller parameter

3) Perturbation in all process parameters and any two controller parameters

The PID controller designed for FOPTD and SOPTD systems is nonfragile for all parameter variation i.e., up to +25% variation all process parameters and controller parameters. The nonfragile nature of PID controller is observed for all the three tuning rules used in this work. There are two important observations when perturbations exist in all process parameters and any controller parameter. The controller is fragile for perturbation in K_c and perturbations in K_c and T_d and it is nonfragile for perturbations in other combinations of the controller parameters. Hence, care should be taken while choosing K_c and T_d values as they make the controller fragile when there are perturbations in both process parameters and controller parameters. This work can be extended to integer and fractional order controllers designed for both stable and unstable systems.

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